



# Probabilistic Model Checking with PRISM

Past, Present & Future

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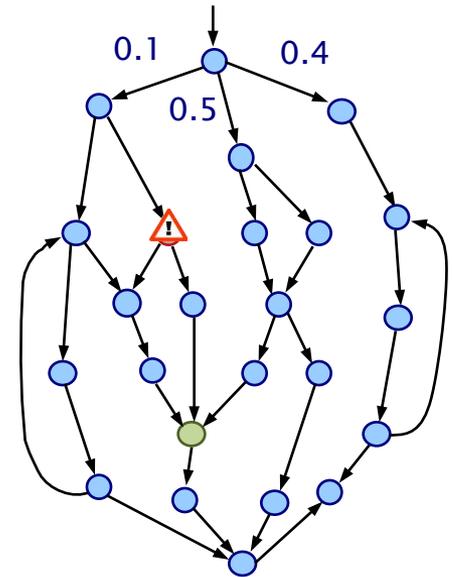
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# Outline

- Probabilistic model checking and PRISM
- Themes and trends
- Advances and applications
- Current research topics
- Challenges & future directions

# Probabilistic model checking

- Construction and analysis of probabilistic models
  - **probability**: failures, uncertainty, noise, randomisation, ...
  - **time**: delays, time-outs, failure rates, ...
  - **costs**: energy, resources, ...
- Quantitative correctness properties expressed in temporal logic, e.g.:
  - **trigger**  $\rightarrow P_{\geq 0.999} [ F^{\leq 20} \text{deploy} ]$
  - “the probability of the airbag deploying within 20 milliseconds of being triggered is at least 0.999”
  - reliability, timeliness, performance, efficiency, ...



# PRISM



- A (brief) history

- late 80s, early 90s: first underlying theory developed
- 2001: first official public release of PRISM
- 2011: version 4.0 - probabilistic real time systems
- 2013: PRISM-games – stochastic multi-player games

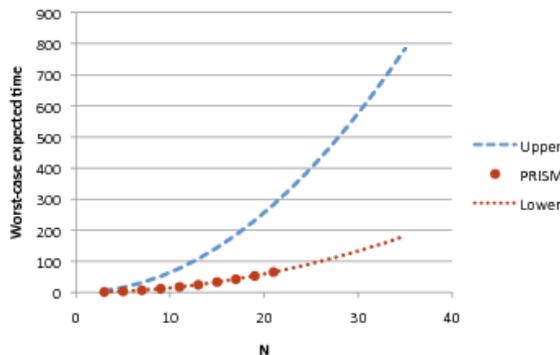
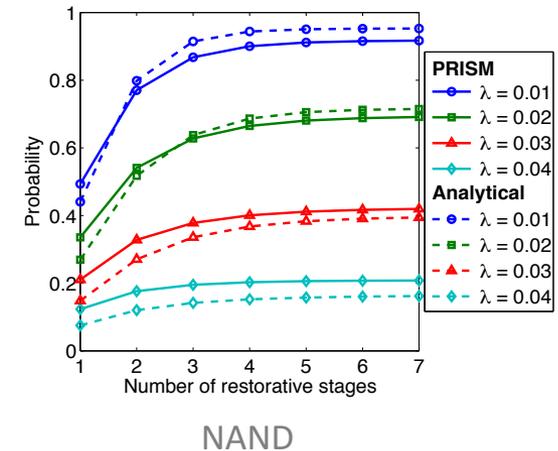
- PRISM today

- used in 100+ institutions; 50,000+ downloads
- broadly applicable; many diverse use cases
- many non-expert (and non-CS) users
- 300 external papers (no involvement from PRISM team)
- flaws found in real-systems; industrial usage

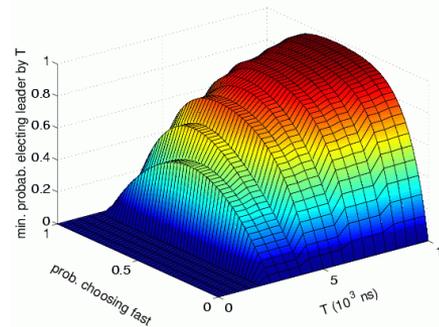


# What can we do with PRISM?

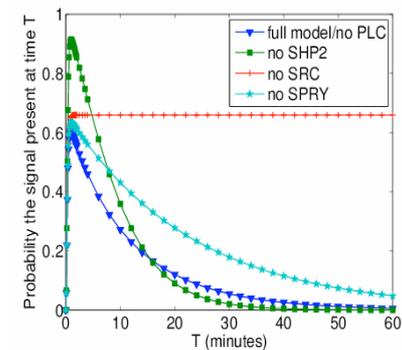
- Identify flaws in existing analyses
  - e.g. reliability of NAND multiplexing
- Investigate conjectures/models
  - e.g. Herman's self-stabilisation
  - e.g. FireWire root contention
  - e.g. cell signalling pathways (FGF)



Herman



FireWire



FGF

# Themes and trends

- Themes in the development of PRISM
  - **theory-to-practice** (and practice-to-theory)
  - wide **collaboration** (theory, algorithms, case studies)
  - **open source** software (and data)
  - overlaps/interacts with many **other disciplines**
- Trends
  - improvement in **scalability** to larger models
  - increasingly **expressive**/powerful classes of model
  - from verification problems to **control** problems
  - ever widening range of **application domains**

# Trends

## Models

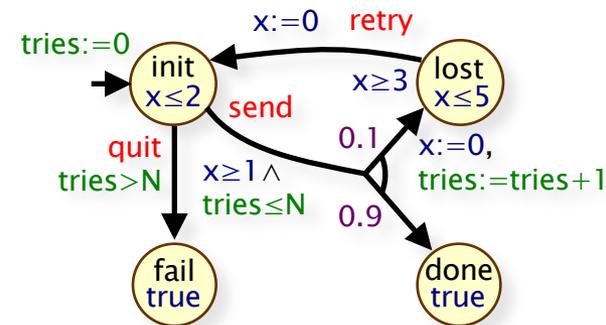
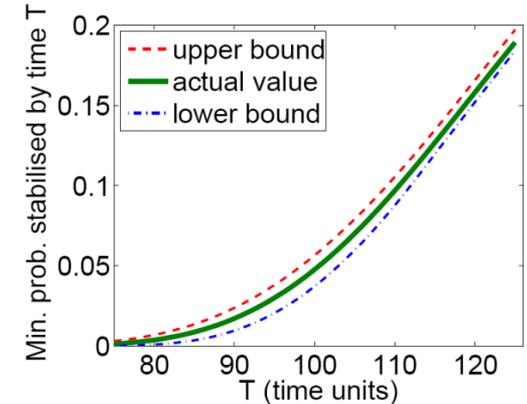
- 
- discrete-time Markov chains
  - probabilistic automata
  - continuous-time Markov chains
  - Markov decision processes
  - probabilistic timed automata
  - stochastic multi-player games
  - ...

## Application domains

- 
- randomised distributed algorithms
  - network/communication protocols
  - computer security
  - performance/reliability
  - systems biology
  - DNA computing
  - robotics & autonomous vehicles
  - wearable/implantable devices
  - ...

# Enabling technologies

- Symbolic model checking
  - [TACAS'00] [TACAS'02] [STTT'04] [CAV'06] ...
- Real-time probabilistic verification
  - [TCS'02] [FMSD'06] [Info&Comp'07] [FORMATS'09] ...
- Quantitative abstraction refinement
  - [QEST'06] [VMCAI'09] [FMSD'10] [QEST'11] ...
- Compositional verification
  - [TACAS'10] [QEST'10] [FASE'11] [Info&Comp'13] ...
- And more...
  - statistical model checking, symmetry reduction, bisimulation minimisation, ...

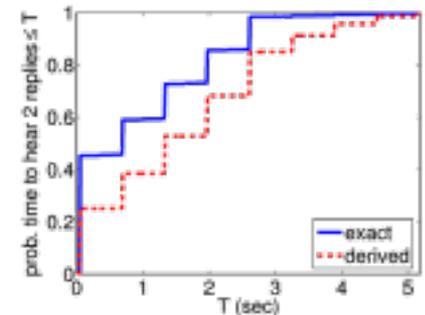


$$\frac{M_1 \models \langle A \rangle_{\geq q} \quad \langle A \rangle_{\geq q} M_2 \langle G \rangle_{\geq p}}{M_1 \parallel M_2 \models \langle G \rangle_{\geq p}}$$

# Case study: Bluetooth

- Device discovery between a pair of Bluetooth devices
  - performance essential for this phase
- Detailed model from official specification
  - two asynchronous 28-bit clocks
  - pseudo-random hopping between 32 frequencies
  - random waiting scheme to avoid collisions
  - 32 Markov chains, over  $3 \times 10^{10}$  states each
  - 17,179,869,184 initial configurations
- Symbolic probabilistic model checking
  - “worst-case expected discovery time is at most 5.17s”

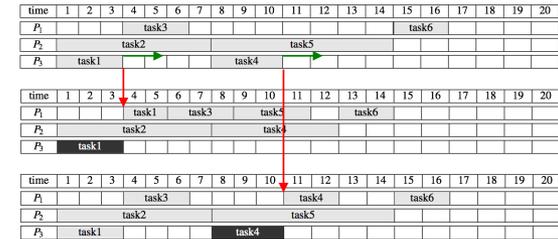
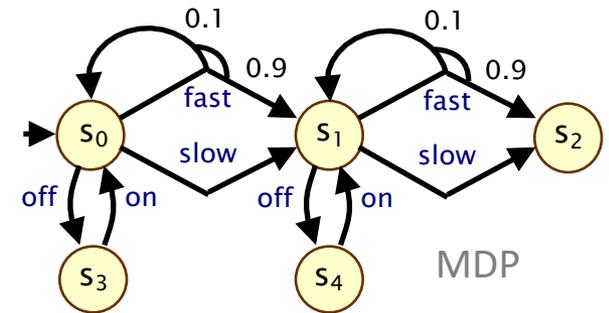
$$\text{freq} = [\text{CLK}_{16-12} + k + (\text{CLK}_{4-2,0} - \text{CLK}_{16-12}) \bmod 16] \bmod 32$$



# Strategy/controller synthesis

- Verification vs. control

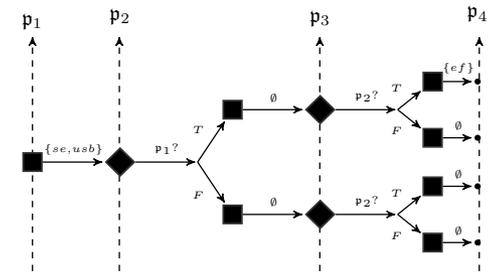
- **verify** that a system is “correct”, for any environment/adversary/... (counterexample yields flaw/attack/...)
- **synthesise** a “correct-by-construction” controller from formal specification (witness yields strategy/controller)



Task schedule [FMSD'13]

- Applications

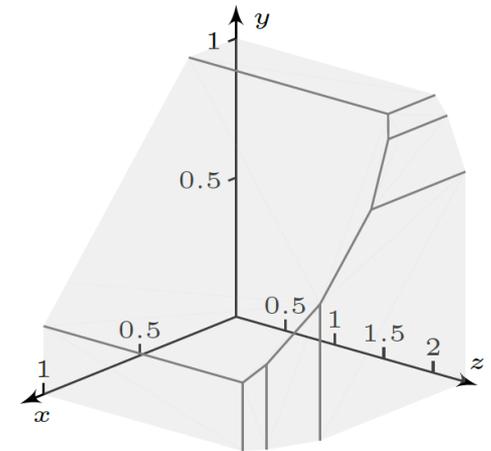
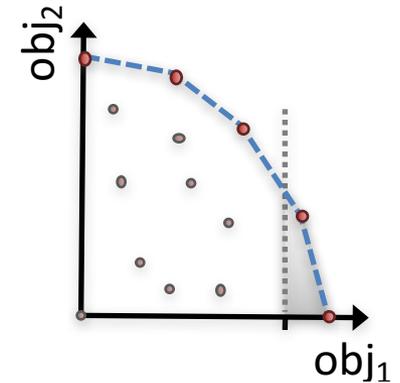
- dynamic power management, robots/autonomous vehicle navigation, task/network scheduling, security, ...



Attack-defence tree [CSF'16]

# Multiple objectives

- **Multi-objective controller synthesis** [LMCS'08] [TACAS'11]
  - trade-offs between conflicting objectives
- **Mix of optimisation and guarantees**
  - e.g. “what strategy **maximises** probability of message transmission, whilst **guaranteeing** expected battery life-time is  $> 10$  hrs?”
  - **Pareto curve** generation/approximation
- **Extensions**
  - permissive controller synthesis of multi-strategies for MDPs [LMCS'15]
  - multiple objectives for multi-player games (see later)



# Robots & autonomous systems

- Navigation for mobile service robots
  - learnt probabilistic navigation maps
  - LTL task specifications + controller synthesis
  - ROS-based runtime planning implementation
  - multi-objective probabilistic guarantees on task completion/duration [IROS'14/IJCAI'15/CDC'16]



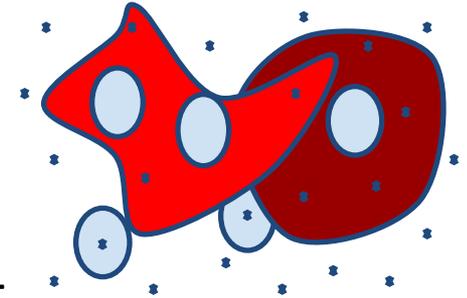
- Autonomous underwater vehicle navigation
  - incremental/parametric verification + controller synthesis
  - probabilistic programming + machine learning to generate realistic component/environment models at runtime

# Parameter synthesis

- Synthesising models that are guaranteed to satisfy quantitative correctness properties is difficult
  - but we can synthesise **controllers** and **parameters**
- Parameter synthesis
  - given a **parametric** model and a property  $\phi$ ...
  - find the optimal parameter values, with respect to an objective function  $O$ , such that the property  $\phi$  is satisfied, if such values exist
- Quantitative parameter synthesis
  - **parameters**: timing delays, rates
  - **objectives**: optimise probability, reward/volume

# Quantitative parameter synthesis

- Timed/hybrid automata
  - find optimal **timing delays** [EMSOFT2014] [HSB'15] [HSCC'16]
  - constraint solving, discretisation + sampling
- Probabilistic timed automata
  - find delays to optimise **probability** [RP2014]
  - parametric symbolic abstraction-refinement
- Continuous-time Markov chains
  - find optimal **rates** [CMSB'14] [ActaInf'16], PRISM-PSY [TACAS'16]
  - constraint solving, uniformisation + sampling
- Focus: practical implementation, real-world usage





# Mobile autonomy challenge

- Autonomous systems
  - **interact** with their environment, which is possibly **adversarial**
  - have **goals/objectives**, which may **conflict**
  - take **decisions**
- Model as **stochastic games**
  - well known from, e.g., decision making in economics
  - many application domains: security, energy grid, etc
- Tool PRISM-games, extension of PRISM [TACAS'16]



# Stochastic multi-player games

- Probabilistic **temporal logic** with coalitions

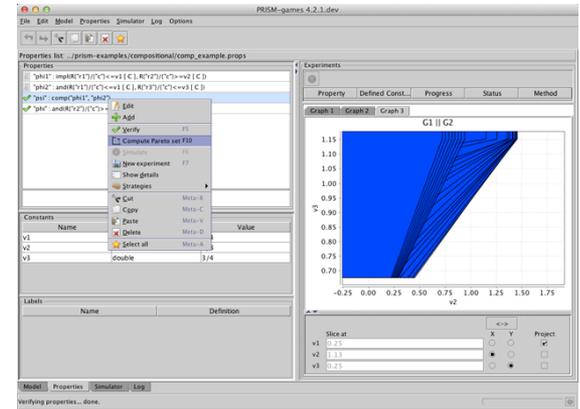
- probabilities, rewards (reachability, total, mean-payoffs/ratios, ...)  
[FMSD'13] [ICALP'16] [ECC'16]

- **Multi-objective** strategy synthesis

- Pareto set computation and optimal achievable trade-offs [MFCS'13] [QEST'13] [TACAS'15]

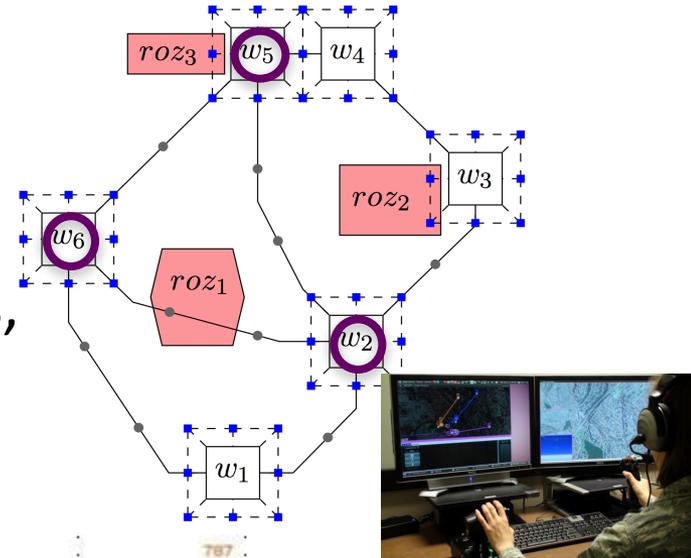
- **Compositional** strategy synthesis [CONCUR'14] [Inf & Comp'16]

- assume-guarantee + multi-objective strategy synthesis
- e.g. local strategies for  $G_1 \models \phi_A$ ,  $G_2 \models \phi_A \Rightarrow \phi_B$   
compose to a global strategy for  $G_1 || G_2 \models \phi_B$

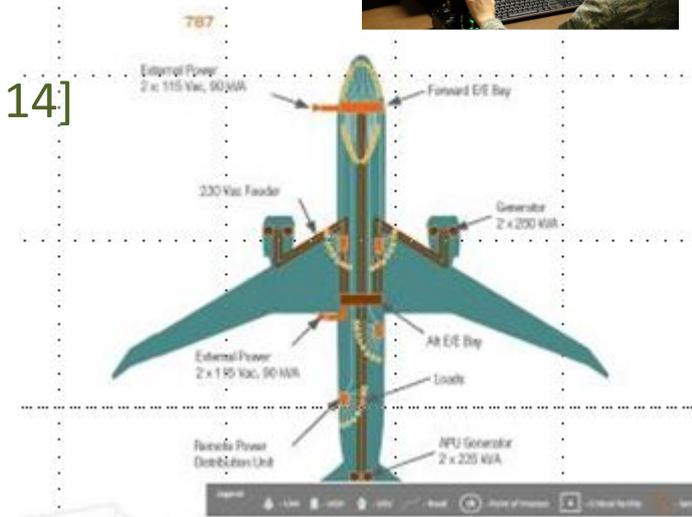


# Applications

- UAV path planning [ICCPs'15]
  - human operator + low-level piloting
  - **quantitative mission objectives**: minimise time/fuel, restricted zones, operator fatigue/workload
  - **multi-objective** MDPs, stoch. games



- Aircraft power distribution [CONCUR'14]
  - **compositional** strategy synthesis in stochastic games (PRISM-games)
  - specify control objectives in LTL using **mean payoff**



# Are games sufficient?

- Complex decisions!
  - goals
  - perception
  - situation awareness
  - context (social, regulatory)
- What about social subtleties?
- What to do in emergency?
  - **moral** decisions, **handover** to driver, obey traffic rules
- Need to make robots human-like...
  - need multi-modal communication, **cognitive reasoning**, trust, ethics, ...

Humans are pretty good at guessing what others on the road will do. Driverless cars are not—and that can be exploited.



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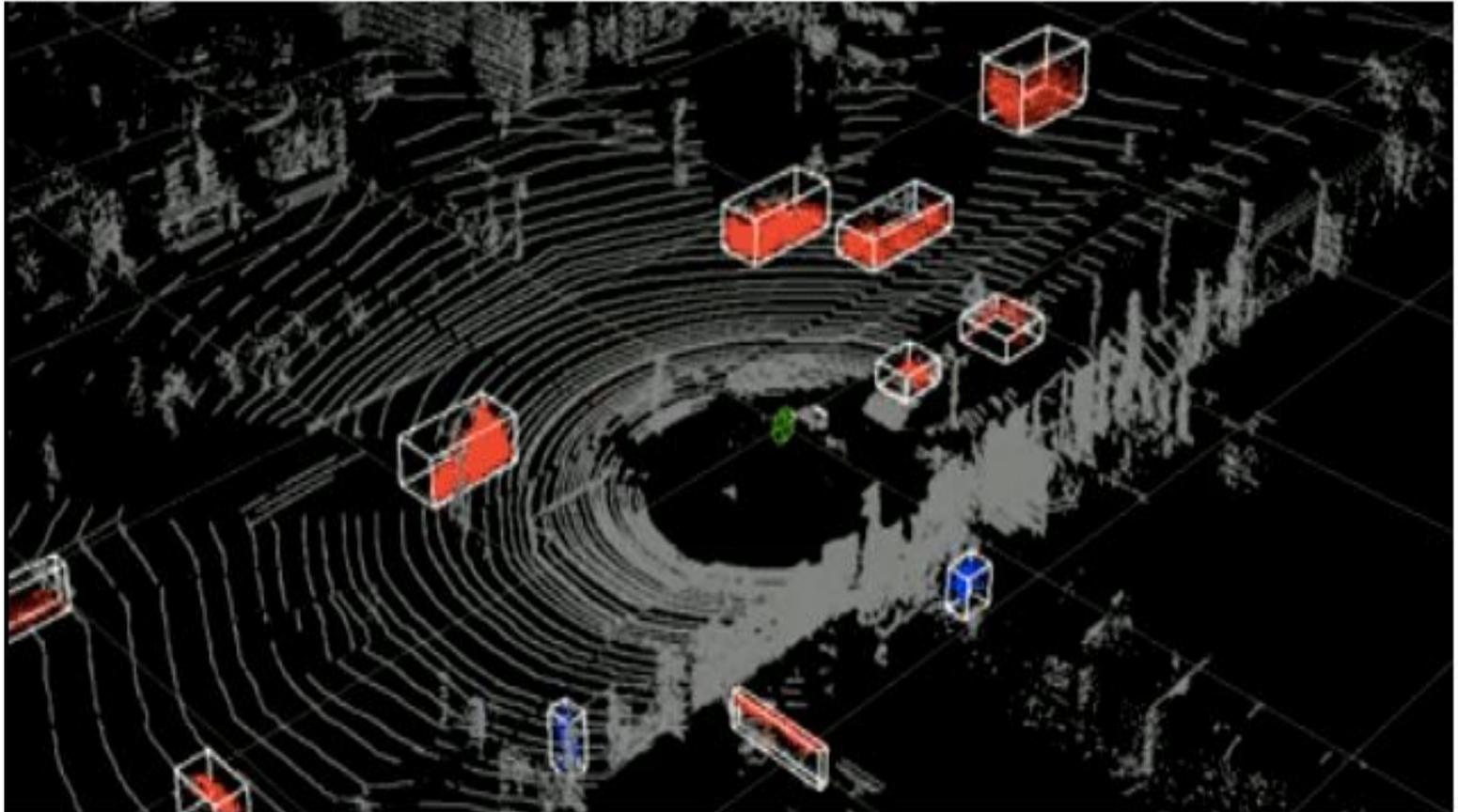
By Samuel English Anthony



# Quantitative verification for trust?

- Social trust: fundamental for mobile autonomy [LK16b]
  - influenced by **external** factors, such as social norms
  - also **internal**: personality, motivation, preferences
  - subjective: would you trust an autonomous taxi to take your child to school?
- Formulate a **temporal logic** to express X's trust in Y for G, based on probabilistic belief [HK17]
- Admits a **model checking** procedure, which can:
  - be used in **decision-making** for robots
  - **explain** decisions, i.e. who is accountable for the action

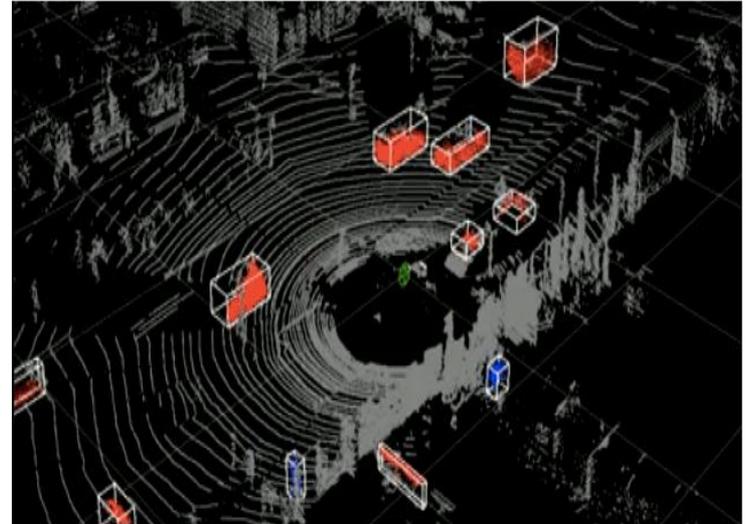
# Perception software



Credits: Oxford Robotics Institute

# Things that can go wrong...

- ...in **perception** software
  - sensor failure
  - object detection failure
- **Machine learning software**
  - not clear how it works
  - does **not** offer guarantees
- **Verification for machine learning?**
  - some progress towards safety verification for neural networks



# Personalisation challenge

- Device must **adapt** to physiology of human wearer
  - achieved through model parameterisation
  - parameter **estimation**, optimal parameter **synthesis**
- **Multiple uses**
  - **automation** of personalised medical intervention
  - device **safety** assurance, for testing
  - reproduce the unique characteristics for **authentication**
- **Focus on ECG based devices**
  - pacemaker models, heart models, synthetic ECGs
  - future work on anxiety monitoring and control

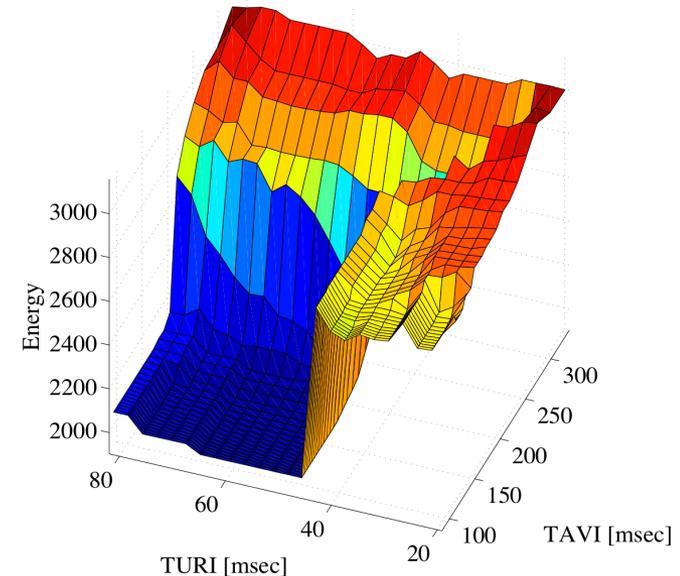
# Pacemaker verification/optimisation

- Hybrid model-based framework
  - timed automata model for pacemaker
  - hybrid heart models in Simulink (non-linear ODEs)



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- Properties
  - (basic safety) maintain 60-100 beats per minute
  - optimisation of energy usage & cardiac output [HSB'16] [HSCC'16]
  - in-silico analysis of rate-adaptive pacemakers [ICHI'14]
  - hardware in the loop [EMBC'15]

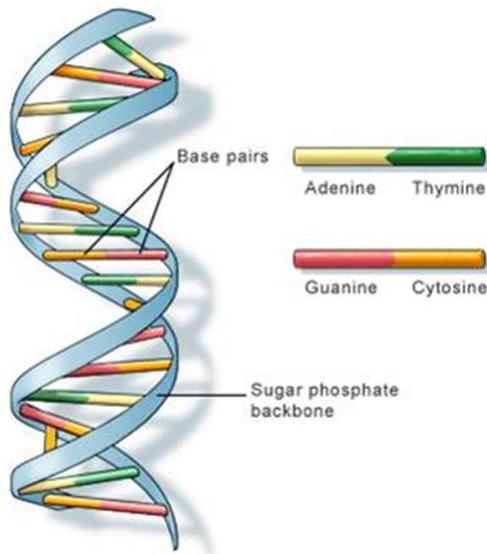


# DNA computation challenge

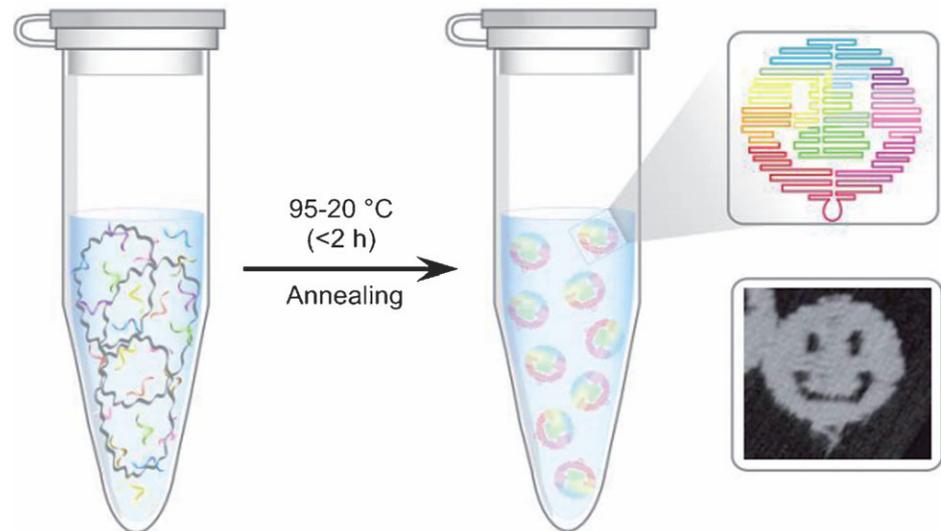
- Moore's law, hence need to make devices smaller...
- **DNA computation**, directly at the molecular level
  - DNA logic circuit designs & programmable nanorobotics
  - asynchronous DNA circuit designs [DNA'16]
- Many applications envisaged
  - e.g. bio-sensing, point of care diagnostics, ...
- Apply quantitative verification and synthesis to
  - find **design flaws** in DNA computing devices [JRSI'12]
  - analyse **reliability and performance** of molecular walkers
  - automatically **synthesise** reaction rates **to guarantee** a specified level of reliability

# DNA nanostructures

- **DNA origami** [Rothemund, *Nature* 2006]
  - DNA can self-assemble into structures – “**molecular IKEA?**”
  - **programmable** self-assembly (can form tiles, nanotubes, boxes that can open, etc.)



U.S. National Library of Medicine

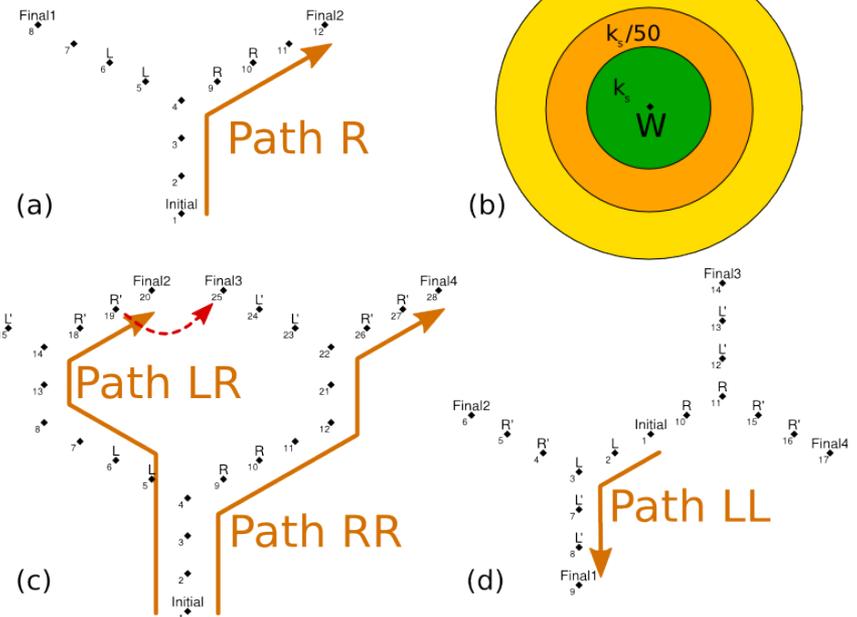


# DNA walker circuits

- Computing with DNA walkers [NatComp'14]

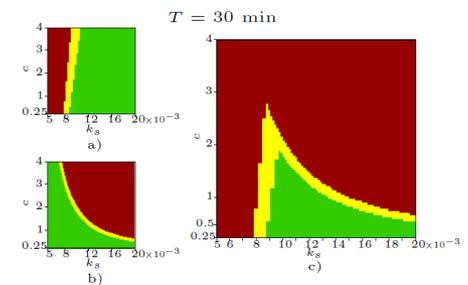
- branching tracks laid out on DNA origami tile
- starts at 'initial', signals when reaches 'final'
- can control 'left'/'right' decision
- any Boolean function

Decision circuits



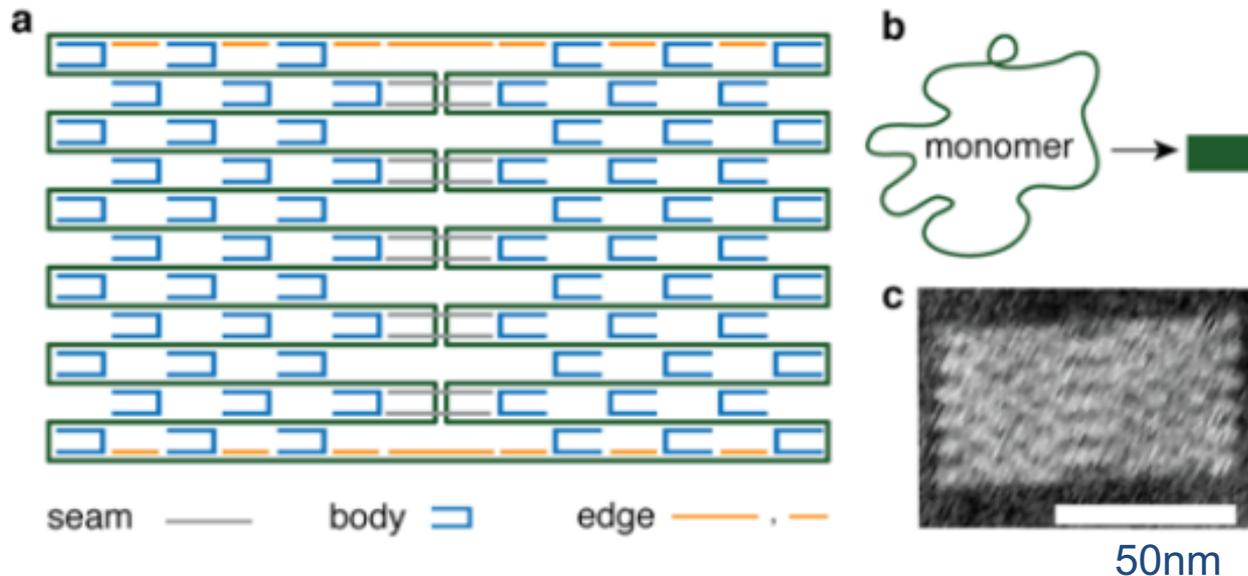
- Parameter synthesis of rates

- for guaranteed reliability level [CMSB'14]



# DNA origami tiles

- DNA origami tiles



- Aim: understand how to control the folding pathway
  - formulate an abstract **Markov chain** model
  - yields **predictions**; perform a range of experiments, consistent with predictions [Nature'15]

# Conclusions

- Probabilistic model checking & PRISM
  - 15 years since first official tool release
  - significant advances in underlying theory & technologies
  - successfully deployed in many application domains
- Many research challenges and applications ahead
  - verification, synthesis, learning, trust, cognitive models, ...
  - autonomous systems, DNA computing, personalised wearable/implantable devices, ...



<http://www.prismmodelchecker.org/>

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(and many more collaborators on case studies & projects)

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