

Description Logic: A Formal Foundation for Ontology Languages and Tools

Part 2: Tools

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Contents

- Motivation for Description Logic reasoning
- Basic reasoning tasks/problems
- Reasoning techniques
 - Tableau
 - Completion
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 - Rule-based
- Other reasoning tasks
- Recent and future work



Description Logic Reasoning





What Are Description Logics?

- Modern DLs (after Baader et al) distinguished by:
 - Fully fledged logics with **formal semantics**
 - **Decidable fragments of FOL** (often contained in C_2)
 - Closely related to Propositional Modal/Dynamic Logics & Guarded Fragment
 - Computational properties well understood (worst case complexity)
 - Provision of **inference services**
 - **Practical** decision procedures (algorithms) for key problems (satisfiability, subsumption, query answering, etc)
 - Implemented **systems** (highly optimised)





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Why Ontology Reasoning?

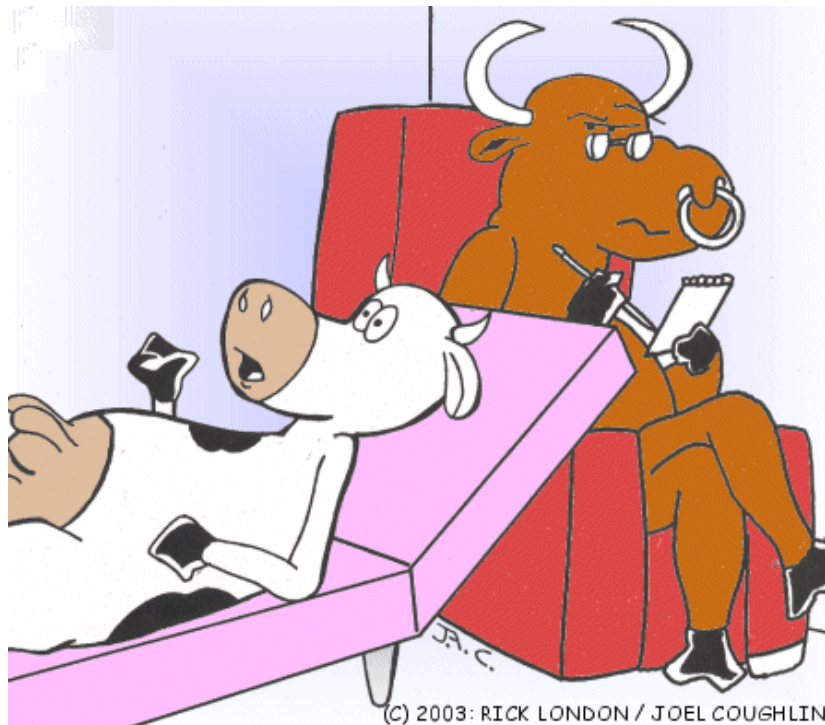
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Why Ontology Reasoning?

- Developing and maintaining quality ontologies is *hard*
- Reasoners allow domain experts to check if, e.g.:
 - classes are consistent (no “obvious” errors)



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Concise Format Abstract Syntax ▶

OWL-Class: [mad+cow](#)
Unsatisfiable concept

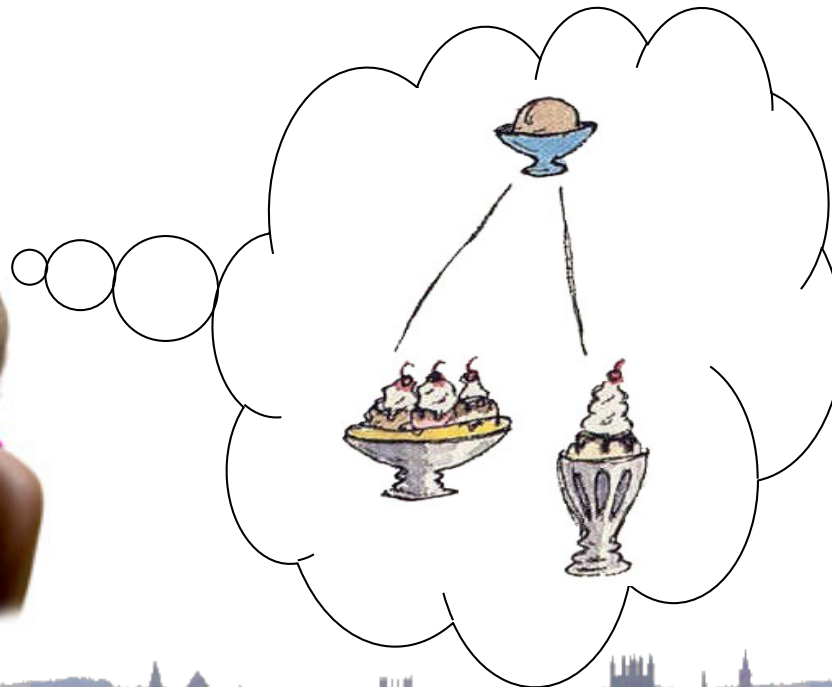
Intersection of:
[\(∃eats . \(\(∃part+of . sheep\) ∩ brain\)\)](#)
[cow](#)

Equivalent to:
[owl:Nothing](#) (Why?)



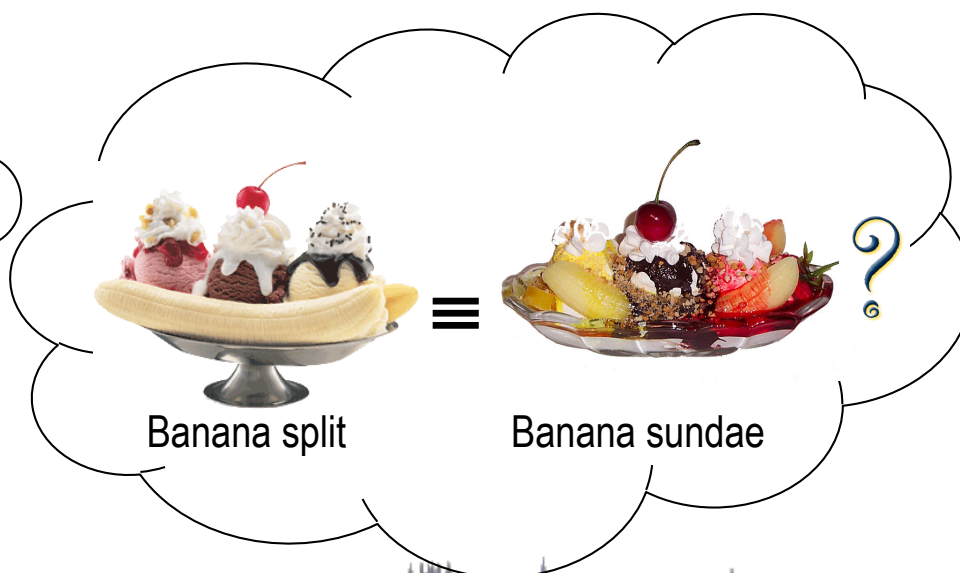
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- Reasoners allow domain experts to check if, e.g.:
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 - expected subsumptions hold (consistent with intuitions)



Why Ontology Reasoning?

- Developing and maintaining quality ontologies is *hard*
- Reasoners allow domain experts to check if, e.g.:
 - classes are consistent (no “obvious” errors)
 - expected subsumptions hold (consistent with intuitions)
 - unexpected equivalences hold (unintended synonyms)





Basic Reasoning Tasks

- **Using ontologies** in applications is also very challenging
 - TBox (schema) may be **large**
 - Abox (data) may be **very large**
 - Query answers may depend on **interactions between schema & data**
- **Query answering**
 - Is the parent of a Doctor necessarily a HappyParent? (schema)
 - Is John a HappyParent? (schema + data)
 - Retrieve all instances of Wizards having pet Owls (schema + data)





Basic Reasoning Problem

- Is an axiom/fact **entailed** by ontology/KB
 - Ontology contains **obvious errors**
 $\mathcal{K} \models C \equiv \perp$ for some concept name C ?
 - Ontology is **consistent with intuitions**
 $\mathcal{K} \models C \sqsubseteq D$ s.t. expert believes $C \not\sqsubseteq D$?
 $\mathcal{K} \models C \not\sqsubseteq D$ or $\mathcal{K} \models C \sqsubseteq D$ s.t. expert believes $C \sqsubseteq D$?
 - Ontology entails **unexpected equivalences**
 $\mathcal{K} \models C \equiv D$ for concept names C and D ?
 - Ontology entails **query answers**
 $\mathcal{K} \models (\text{Parent} \sqcap \exists \text{hasChild.Doctor}) \sqsubseteq \text{HappyParent}$?
 $\mathcal{K} \models \text{John:HappyParent}$?
Retrieve all individuals a s.t. $\mathcal{K} \models a:(\text{Wizard} \sqcap \exists \text{hasPet.Owl})$



Reasoning Techniques

- **Direct**
 - Specially designed reasoning algorithms
 - Operate on the DL (more or less) directly
- **Indirect**
 - Translate into some equivalent problem in another formalism
 - Solve resulting problem using appropriate technology





Direct Reasoning Techniques

- Two basic classes of algorithm
 - **Model construction**
 - Prove entailment does not hold by constructing model of KB in which axiom/fact is false
 - E.g., tableau algorithms
 - tableau expansion rules used to derive **new ABox facts**
 - **Proof derivation**
 - Prove entailment holds by deriving axiom/fact from KB
 - E.g., structural, completion, rule-based algorithms
 - deduction rules used to derive **new TBox axioms**





Tableau Algorithms

- Currently the most **widely used** technique
 - Basis for reasoners such as FaCT++, HermiT, Pellet, Racer, ...
- Mainly used with more expressive logics (e.g., **OWL**)
 - Standard technique is to negate premise axiom/fact
 - HyperTableau may also work well with sub-Boolean DLs
- Most effective for **schema reasoning**
 - Large datasets may necessitate construction of large models
 - Query answering may require each possible answer to be checked
 - Optimisations can limit but not eliminate these problems





Tableau Algorithms

- Transform entailment to **KB (un)satisfiability**
 - $\mathcal{K} \models a:C$ iff $\mathcal{K} \cup \{a:(\neg C)\}$ is *not* satisfiable
 - $\mathcal{K} \models C \sqsubseteq D$ iff $\mathcal{K} \cup \{a:(C \sqcap \neg D)\}$ is *not* satisfiable (for new a)
- Start with **facts** explicitly asserted in ABox
 - e.g., John:HappyParent, John hasChild Mary
- Use **expansion rules** to derive new **ABox facts**
 - e.g., John:Parent, John: \forall hasChild.(Doctor \sqcup \exists hasChild.Doctor)
- Construction fails if obvious contradiction (**clash**)
 - e.g., Mary:Doctor, Mary: \neg Doctor



Expansion Rules for \mathcal{ALC}

- \sqcap -rule: if 1. $a : (C_1 \sqcap C_2) \in \mathcal{A}$, and
2. $\{a : C_1, a : C_2\} \not\subseteq \mathcal{A}$
then set $\mathcal{A}_1 = \mathcal{A} \cup \{a : C_1, a : C_2\}$
- \sqcup -rule: if 1. $a : (C_1 \sqcup C_2) \in \mathcal{A}$, and
2. $\{a : C_1, a : C_2\} \cap \mathcal{A} = \emptyset$
then set $\mathcal{A}_1 = \mathcal{A} \cup \{a : C_1\}$ and $\mathcal{A}_2 = \mathcal{A} \cup \{a : C_2\}$
- \exists -rule: if 1. $a : (\exists S.C) \in \mathcal{A}$, and
2. there is no b such that $\{a, b\} : S, b : C \subseteq \mathcal{A}$,
then set $\mathcal{A}_1 = \mathcal{A} \cup \{a, d\} : S, d : C\}$, where d is new in \mathcal{A}
- \forall -rule: if 1. $\{a : (\forall S.C), a, b\} : S \subseteq \mathcal{A}$, and
2. $b : C \notin \mathcal{A}$
then set $\mathcal{A}_1 = \mathcal{A} \cup \{b : C\}$

- some rules are **nondeterministic**, e.g., \sqcup , \leq
- implementations use **backtracking** search





Expansion Example

$\mathcal{T} = \{\text{Doctor} \sqsubseteq \text{Person}, \text{Parent} \equiv \text{Person} \sqcap \exists \text{hasChild}.\text{Person},$
 $\text{HappyParent} \equiv \text{Parent} \sqcap \forall \text{hasChild}.\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor}\}$

$\mathcal{A} = \{\text{John}:\text{HappyParent}, \text{John hasChild Mary}\}$

$\models \text{Mary}:\text{Doctor}$?





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John:HappyParent, John hasChild Mary





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John:HappyParent, John hasChild Mary

Mary:¬Doctor





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John:HappyParent, John hasChild Mary

Mary: \neg Doctor

John:Parent, John: \forall hasChild.(Doctor \sqcup \exists hasChild.Doctor)





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John:Person, John: \exists hasChild.Person

Mary:(Doctor \sqcup \exists hasChild.Doctor)





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Mary:(Doctor \sqcup \exists hasChild.Doctor)

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- John:HappyParent, John hasChild Mary
- ✗ Mary:¬Doctor
- John:Parent, John:∀hasChild.(Doctor ⊔ ∃hasChild.Doctor)
- John:Person, John:∃hasChild.Person
- Mary:(Doctor ⊔ ∃hasChild.Doctor)
- John hasChild a, a:Person, a:(Doctor ⊔ ∃hasChild.Doctor)
- ✗ Mary:Doctor



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John:Person, John: \exists hasChild.Person

Mary:(Doctor \sqcup \exists hasChild.Doctor)

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Mary: \exists hasChild.Doctor

Mary hasChild b, b:Doctor, b:Person



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?



John:HappyParent, John hasChild Mary, Mary: $\forall \text{hasChild}.\perp$

Mary: $\neg \text{Doctor}$

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John:Person, John: $\exists \text{hasChild}.\text{Person}$

Mary: $\text{(Doctor} \sqcup \exists \text{hasChild}.\text{Doctor)}$

John hasChild a, a:Person, a: $\text{(Doctor} \sqcup \exists \text{hasChild}.\text{Doctor)}$

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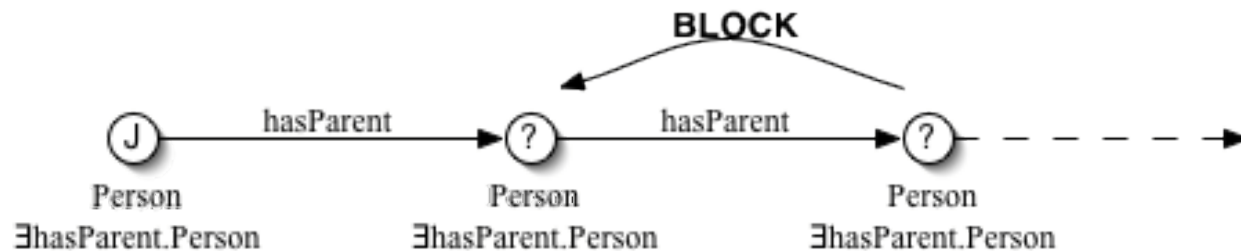
Mary hasChild b, b:Doctor, b:Person

\times b: \perp



Termination

- Simplest DLs are naturally terminating
 - Rules produce strictly smaller concepts
- Most DLs require some form of **blocking**
 - E.g., $\{\text{Person} \sqsubseteq \exists \text{hasParent}.\text{Person}, \text{John}:\text{Person}\}$



- Expressive DLs need more complex blocking



Correctness

A **decision procedure** for KB satisfiability

Will **always give an answer**, and will **always give the *right* answer** i.e., it is correct (sound and complete) and terminating

Sound: if clash-free ABox is constructed, then KB is satisfiable

Given fully expanded clash-free ABox, we can trivially construct a model

Complete: if KB is satisfiable, then clash-free ABox is constructed

Given a model, we can use it to guide application of non-deterministic rules

Terminating: the algorithm will always produce an answer

Upper bound on number of new individuals we can create,
so ABox construction will always terminate





Highly Optimised Implementations

- Lazy unfolding
- Simplification and rewriting
 - Absorption: $A \sqcap B \sqsubseteq C \longrightarrow A \sqsubseteq C \sqcup \neg B$
- Detection of tractable fragments (\mathcal{EL})
- Fast semi-decision procedures
 - Told subsumer, model merging, ...
- Search optimisations
 - Dependency directed backtracking
- Reuse of previous computations
 - Of (un)satisfiable sets of concepts (conjunctions)
- Heuristics
 - Ordering don't know and don't care non-determinism



Completion Algorithms

- **Newer technique**, but gaining in popularity
 - Basis for reasoners such as CEL, snorocket, CB, ...
- Mainly used with less expressive logics (e.g., **OWL 2 EL**)
 - Usually restricted to deterministic fragments (e.g., no disjunction)
 - But newer methods may be able to deal with nondeterminism
- Effective with **very large schemas**
 - Polynomial time algorithms for Horn DLs (such as OWL 2 EL)
 - Finds all subsumption relations in a single computation
- Also effective with **very large data sets**
 - Polynomial in the size of the data
 - New techniques exploit DB technology for scalability



Completion Algorithms

- Transform KB axioms into **simplified form**
 - e.g., $C \sqsubseteq \exists R.(C \sqcap D) \rightsquigarrow C \sqsubseteq \exists R.A, A \sqsubseteq C \sqcap D$
- Use **completion rules** to derive new **TBox axioms**

e.g., ProudParent $\sqsubseteq \exists \text{hasChild}.\text{Doctor}$,
Doctor $\sqsubseteq \text{Person}$,
 $\exists \text{hasChild}.\text{Person} \sqsubseteq \text{Parent}$
 \rightsquigarrow ProudParent $\sqsubseteq \text{Parent}$

- **Structural algorithms** used with early DLs can be seen as naïve (and typically incomplete) form of completion



Completion Rules for \mathcal{ELH}

$$\frac{A \sqsubseteq B \quad B \sqsubseteq C \in \mathcal{O}}{A \sqsubseteq C}$$

$$\frac{A \sqsubseteq B \quad A \sqsubseteq C \quad B \sqcap C \sqsubseteq D \in \mathcal{O}}{A \sqsubseteq D}$$

$$\frac{A \sqsubseteq B \quad B \sqsubseteq \exists r.C \in \mathcal{O}}{A \sqsubseteq \exists r.C}$$

$$\frac{A \sqsubseteq \exists r.B \quad r \sqsubseteq s \in \mathcal{O}}{A \sqsubseteq \exists s.B}$$

$$\frac{A \sqsubseteq \exists r.B \quad B \sqsubseteq C \quad \exists r.C \sqsubseteq D \in \mathcal{O}}{A \sqsubseteq D}$$



Completion Rules for \mathcal{ELH}

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$$\frac{A \sqsubseteq \exists r.B \quad B \sqsubseteq C \quad \exists r.C \sqsubseteq D \in \mathcal{O}}{A \sqsubseteq D}$$



Correctness

A **decision procedure** for classification

Will **always give an answer**, and will **always give the *right* answer**
i.e., it is correct (sound and complete) and terminating

Sound: if $C \sqsubseteq D$ is derived, then KB entails $C \sqsubseteq D$

Completion rules are locally correct (preserve entailments)

Complete: if $C \sqsubseteq D$ is entailed by KB, then $C \sqsubseteq D$ is derived

Completion rules cover all cases

Terminating: the algorithm will always produce an answer

Upper bound on number of axioms of the form $C \sqsubseteq D$ or $C \sqsubseteq \exists r.D$,
so completion will always “saturate”



Query Rewriting

- Basis for systems such as QuOnto, Owlgres and Quill
- Mainly used with less expressive logics (e.g., [OWL 2 QL](#))
 - Usually restricted to deterministic fragments
 - Axioms may also be asymmetric (different restrictions on lhs/rhs)
- Focus is on [query answering](#)
 - Usually assume that TBox/schema is small and/or simple
- Effective with [very large data sets](#)
 - Rewritings typically produce a Datalog program
 - May even produce union of conjunctive queries (\approx SQL query)
 - Data can be stored/left in relational DB
 - Can delegate query answering to RDBMS



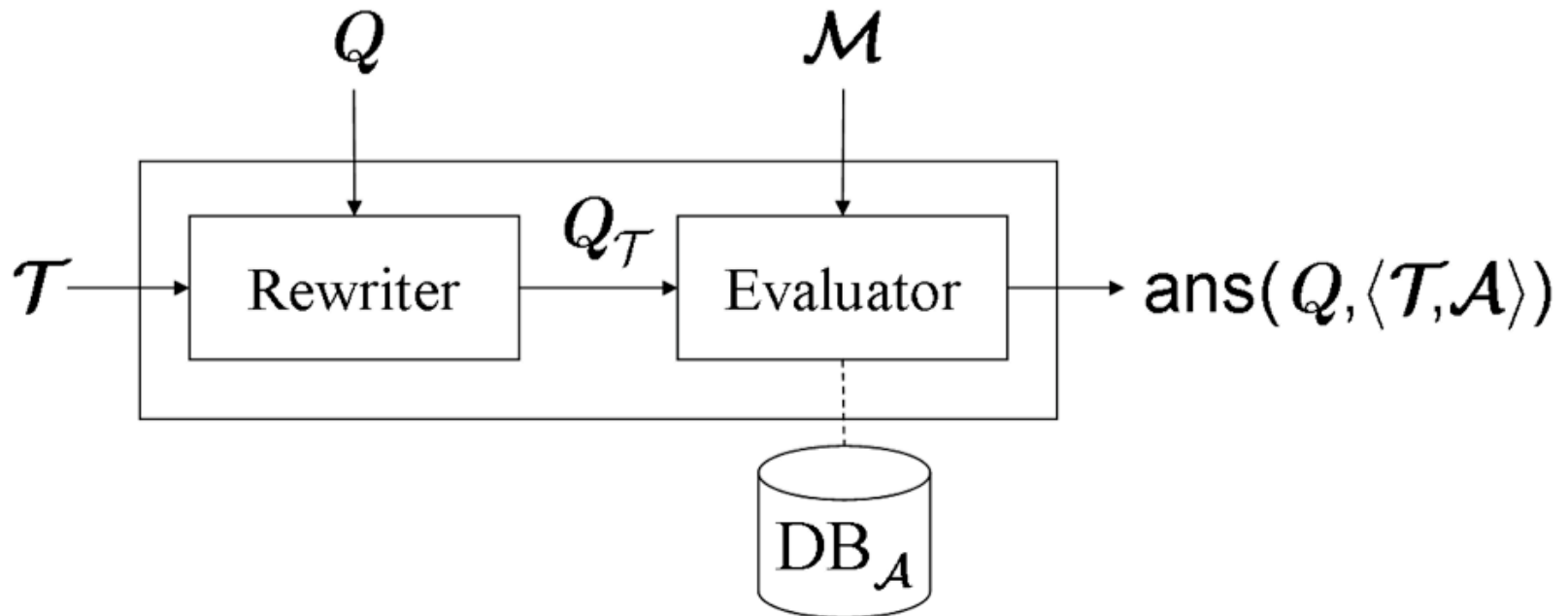


Query Rewriting

- Use KB axioms \mathcal{T} to **expand** query Q to query $Q_{\mathcal{T}}$
 - e.g.,
 $\text{Professor} \sqsubseteq \text{Teacher},$
 $Q(x) \leftarrow \text{Teacher}(x),$
 $\rightsquigarrow Q_{\mathcal{T}}(x) \leftarrow \text{Professor}(x) \cup \text{Teacher}(x)$
- Use mappings to **evaluate** expanded query against DB
 - KB axioms no longer considered (internalised in query)
 - ABox/DB not used in query rewriting
 - Can be used without knowledge of DB contents and/or when access to DB is limited
- Can also use for schema reasoning
 - $C \sqsubseteq D$ iff after adding $a:C$ for new individual a , $\text{KB} \models a:D$



System Architecture





Query Rewriting Example

$\mathcal{T} =$

$\text{Teacher} \sqsubseteq \exists \text{teaches}$
$\text{Professor} \sqsubseteq \text{Teacher}$
$\exists \text{hasTutor}^- \sqsubseteq \text{Professor}$

$Q_0(x) \leftarrow \text{teaches}(x, y)$

$\mathcal{M} =$

$\text{Professor} \mapsto \text{SELECT } 1 \text{ FROM Professor}$
$\text{hasTutor} \mapsto \text{SELECT } 1, 2 \text{ FROM hasTutor}$





Query Rewriting Example

$\mathcal{T} =$

Teacher $\sqsubseteq \exists \text{teaches}$
Professor $\sqsubseteq \text{Teacher}$
$\exists \text{hasTutor}^- \sqsubseteq \text{Professor}$

$Q_0(x) \leftarrow \text{teaches}(x, y)$
 $Q_0(x) \leftarrow \text{Teacher}(x)$
 $Q_0(x) \leftarrow \text{Professor}(x)$
 $Q_0(x) \leftarrow \text{hasTutor}(y, x)$

$\mathcal{M} =$

Professor $\mapsto \text{SELECT } 1 \text{ FROM Professor}$
hasTutor $\mapsto \text{SELECT } 1, 2 \text{ FROM hasTutor}$





Query Rewriting Example

$\mathcal{T} =$

$\text{Teacher} \sqsubseteq \exists \text{teaches}$
$\text{Professor} \sqsubseteq \text{Teacher}$
$\exists \text{hasTutor}^- \sqsubseteq \text{Professor}$

$Q_0(x) \leftarrow \text{teaches}(x, y)$

$Q_0(x) \leftarrow \text{Teacher}(x)$

$Q_0(x) \leftarrow \text{Professor}(x)$

$Q_0(x) \leftarrow \text{hasTutor}(y, x)$

$\mathcal{M} =$

$\text{Professor} \mapsto \text{SELECT } 1 \text{ FROM Professor}$
$\text{hasTutor} \mapsto \text{SELECT } 1, 2 \text{ FROM hasTutor}$

$Q_{\mathcal{T}} =$

$\text{SELECT } 1 \text{ FROM Professor UNION}$
$\text{SELECT } 2 \text{ FROM hasTutor}$





Query Rewriting Example

$\mathcal{T} =$

$\text{Teacher} \sqsubseteq \exists \text{teaches}$
$\text{Professor} \sqsubseteq \text{Teacher}$
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$\text{SELECT } 2 \text{ FROM hasTutor}$

$\text{DB} =$

$\text{Professor} = \{\text{Michael}\}$
$\text{hasTutor} = \{\langle \text{Rob}, \text{Ian} \rangle, \langle \text{Bruno}, \text{Georg} \rangle\}$





Query Rewriting Example

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$\text{DB} =$

$\text{Professor} = \{\text{Michael}\}$
$\text{hasTutor} = \{\langle \text{Rob}, \text{Ian} \rangle, \langle \text{Bruno}, \text{Georg} \rangle\}$

$\text{ans}(Q_0, \langle \mathcal{T}_0, \mathcal{A}_0 \rangle) = \{\text{Michael}, \text{Ian}, \text{Georg}\}$





Correctness

- Rewriting can be shown to be correct
i.e., $\text{ans}(Q, \langle T, \mathcal{A} \rangle) = \text{ans}(Q_T, \langle \emptyset, \mathcal{A} \rangle)$
- Query answer is correct iff system used to compute $\text{ans}(Q_T, \langle \emptyset, \mathcal{A} \rangle)$ is correct
 - e.g., if DBMS is sound complete and terminating





Rule-Based Algorithms

- Basis for systems such as **Oracle's OWL Prime**
 - And widely used to provide *some* OWL support in rule systems
- Mainly used with less expressive logics (e.g., **OWL 2 RL**)
 - Usually restricted to **deterministic** and **existential-free** fragments
 - No disjunction and **cannot infer existence of new individuals**
 - Syntactic restrictions may also be asymmetric
 - e.g., existentials allowed on lhs of axioms, but not on rhs
- Focus is on **query answering**
 - Usually assume that TBox/schema is small and/or simple
- Can be effective with **large data sets**
 - Use rule-extended RDBMS for efficiency



Rule-Based Algorithms

- Rules operate on KB axioms and facts
 - Axioms and facts often in the form of **RDF triples**
 - e.g., $\text{Doctor} \sqsubseteq \text{Person}, \text{John}:\text{Doctor}$
 $\rightsquigarrow \langle \text{Doctor} \text{ rdfs:subClassOf } \text{Person} \rangle, \langle \text{John} \text{ rdf:type } \text{Doctor} \rangle$
- Rules **materialise** implied facts (triples) in **ABox**
e.g.,
 $\langle ?x \text{ rdf:type } ?c_2 \rangle \leftarrow \langle ?c_1 \text{ rdfs:subClassOf } ?c_2 \rangle \wedge \langle ?x, \text{rdf:type}, ?c_1 \rangle$
 $\langle \text{Doctor} \text{ rdfs:subClassOf } \text{Person} \rangle$
 $\langle \text{John} \text{ rdf:type } \text{Doctor} \rangle$
 $\rightsquigarrow \langle \text{John} \text{ rdf:type } \text{Person} \rangle$
- Rules applied until ABox is **saturated**
 - Query answering then reduces to look-up in saturated ABox
 - Can be delegated to DBMS if saturated ABox stored in DB



Rules for OWL RL (*DLP*)

Table 7. The Semantics of Class Axioms

	if	then	
cax-sco	$T(?c_1, \text{rdfs:subClassOf}, ?c_2)$ $T(?x, \text{rdf:type}, ?c_1)$	$T(?x, \text{rdf:type}, ?c_2)$	
cax-eqc1	$T(?c_1, \text{owl:equivalentClass}, ?c_2)$ $T(?x, \text{rdf:type}, ?c_1)$	$T(?x, \text{rdf:type}, ?c_2)$	
cax-eqc2	$T(?c_1, \text{owl:equivalentClass}, ?c_2)$ $T(?x, \text{rdf:type}, ?c_2)$	$T(?x, \text{rdf:type}, ?c_1)$	
cax-dw	$T(?c_1, \text{owl:disjointWith}, ?c_2)$ $T(?x, \text{rdf:type}, ?c_1)$ $T(?x, \text{rdf:type}, ?c_2)$	false	
cax-adc	$T(?x, \text{rdf:type}, \text{owl:AllDisjointClasses})$ $T(?x, \text{owl:members}, ?y)$ $\text{LIST}[?y, ?c_1, \dots, ?c_n]$ $T(?z, \text{rdf:type}, ?c_i)$ $T(?z, \text{rdf:type}, ?c_j)$	false	for each $1 \leq i < j \leq n$

- There are **many** rules
 - This is only one of 9 tables, most of which are *much* larger





Correctness

- Typically **sound but not complete**
- May be complete for certain kinds of KB + query
 - Implementations based **OWL 2 RL rules** will be complete w.r.t. atomic facts, i.e., facts of the form

a:C

a P b

where C is a class name and P is a property



Other Reasoning Services





Other Reasoning Services

- Range of new “non-standard” services supporting, e.g.:
 - **Error diagnosis** and repair

The screenshot shows a software interface for reasoning with OWL classes. At the top, there are buttons for "Concise Format" and "Abstract Syntax", followed by a right-pointing arrow. Below this, the text "OWL-Class: [mad+cow](#)" is displayed. The main window is titled "Explanation" and contains the following text:

Axioms causing the inference
mad+cow = owl:Nothing:

- 1) $(\text{mad+cow} = ((\exists \text{eats} . ((\exists \text{part+of} . \text{sheep}) \cap \text{brain})) \cap \text{cow}))$
- 2) $\perp (\text{sheep} \subseteq \text{animal})$
- 3) $\perp (\text{cow} \subseteq \text{vegetarian})$
- 4) $\perp (\text{vegetarian} = (\text{animal} \cap (\forall \text{eats} . (\neg \text{animal})) \cap (\forall \text{eats} . (\neg (\exists \text{part+of} . \text{animal}))))))$

At the bottom of the explanation window, there is a checkbox labeled "Strike out irrelevant parts of axioms". Below the explanation window, there is a yellow box containing the text "[owl:Nothing](#) (Why?)". In the background, there is a cartoon illustration of a pig's face and a person's torso.





Advanced Reasoning Tasks

- Range of new “non-standard” services supporting, e.g.:
 - **Modular design** and **integration**
 - What is the effect of merging O_2 into O_1 ?
 - **Module Extraction**
 - Extract a (small) module from O capturing all “relevant” information about some concept or set of concepts
 - **Query and Predicate emptiness**
 - Check if query (or query containing given predicate) is empty for *all* ABoxes
 - **Bottom-up design**
 - Find a (small and specific) concept describing a set of individuals



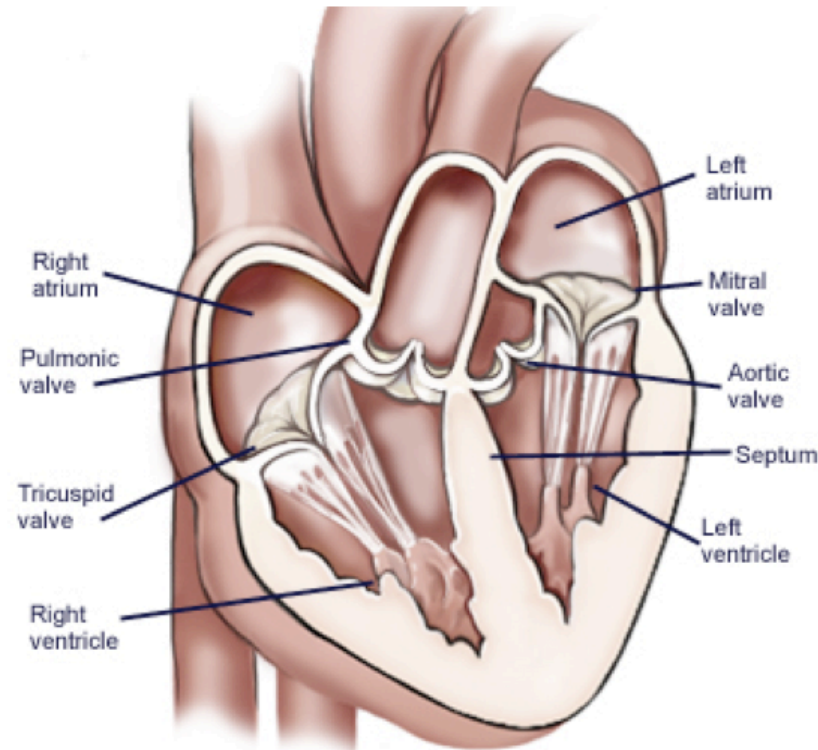
Recent and Future Work





Ontology Languages & Formalisms

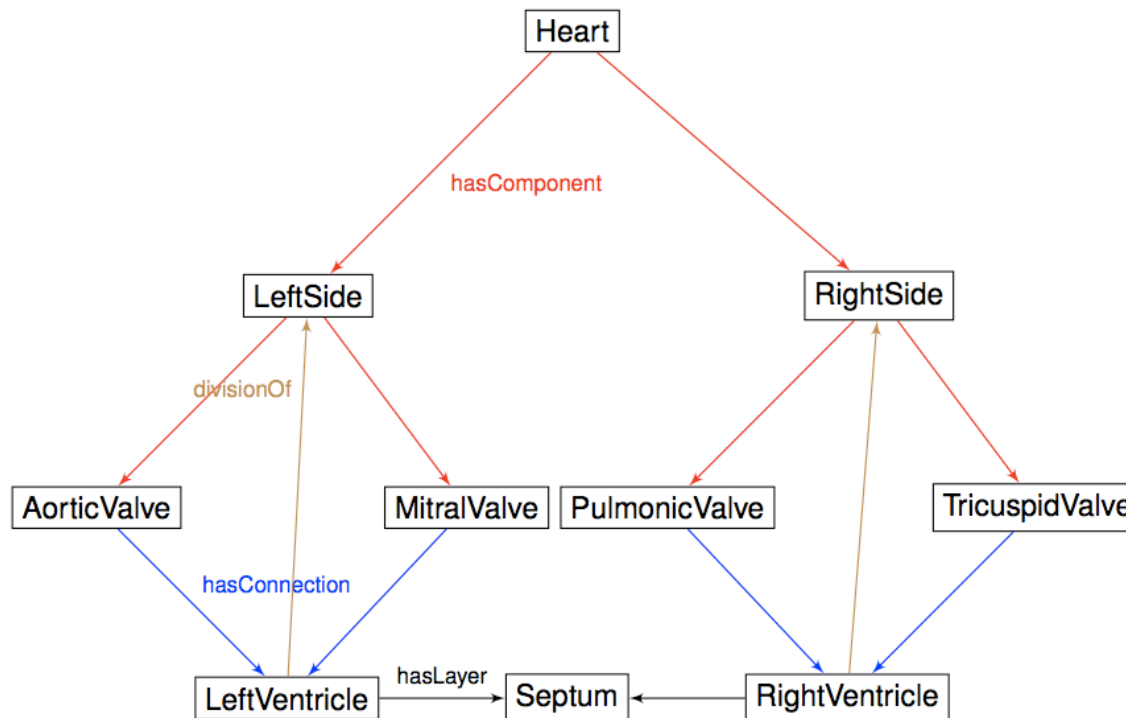
- DLs poor for modelling **non-tree structures**
 - E.g., physically structured objects





Ontology Languages & Formalisms

- DLs poor for modelling **non-tree structures**
 - E.g., physically structured objects





Ontology Languages & Formalisms

- DLs poor for modelling **non-tree structures**
 - E.g., physically structured objects
- **Description graphs** [1] allow for modelling “**prototypes**”
 - Prototypes resemble small ABoxes
 - Reasoning performance may also be significantly improved
 - Some restrictions needed for decidability
 - E.g., on roles used in TBox and in prototypes

[1] Motik, Cuenca Grau, Horrocks, and Sattler. Representing Structured Objects using Description Graphs. In Proc. of KR 2008.





Ontology Languages & Formalisms

- Integration of (expressive) DLs with DBs
 - Open world semantics can be unintuitive
 - Users may want [integrity constraints](#) as well as axioms
 - Reasoning with data can be problematic
 - [Scalability](#) & persistence are both issues
 - Solution could be closer [integration with DBs](#) [1]
 - Challenge is to find a coherent yet practical semantics

[1] Boris Motik, Ian Horrocks, and Ulrike Sattler. Bridging the Gap Between OWL and Relational Databases. In Proc. of WWW 2007.





New Reasoning Techniques

- New **hypertableau** calculus [1]
 - Uses more complex hyper-resolution style expansion rules
 - Reduces non-determinism
 - Uses more sophisticated blocking technique
 - Reduces model size
- New **HermiT** DL reasoner
 - Implements optimised hypertableau algorithm [2]
 - Already outperforms SOTA tableau reasoners

[1] Boris Motik, Rob Shearer, and Ian Horrocks. Optimized Reasoning in Description Logics using Hypertableaux. In Proc. of CADE 2007.

[2] Boris Motik and Ian Horrocks. Individual Reuse in Description Logic Reasoning. In Proc. of IJCAR 2008.





New Reasoning Techniques

- **Completion**-based decision procedures [1]
 - Use proof search rather than model search
 - Crucial “trick” is to use tableau like techniques to guide and restrict derivations
 - Reasoning time for SNOMED reduced by 2 orders of magnitude

[1] Yevgeny Kazakov. Consequence-Driven Reasoning for Horn SHIQ Ontologies. Proc. of IJCAI 2009 (best paper).





New Reasoning Techniques

- “Combined” decision procedures [1]
 - Combination of materialisation and query rewriting
 - Partial saturation of ABox to deal with existentials
 - adds new “representative” individuals
 - Enhanced query rewriting applied to part-saturated ABox
 - Sound and complete for (at least) OWL 2 EL ontologies
 - Early experiments very encouraging w.r.t. scalability

[1] Carsten Lutz, David Toman, and Frank Wolter. Conjunctive Query Answering in the Description Logic EL using a Relational Database System. Proc. of IJCAI 2009.





New Reasoning Services

- Support for **ontology re-use**
 - **Integrate** multiple ontologies [1] and/or **Extract** (small) modules [2]
 - New reasoning problems arise
 - Conservative extension, safety, ..

[1] Bernardo Cuenca Grau, Yevgeny Kazakov, Ian Horrocks, and Ulrike Sattler. A Logical Framework for Modular Integration of Ontologies. In Proc. of IJCAI 2007.

[2] Bernardo Cuenca Grau, Ian Horrocks, Yevgeny Kazakov, and Ulrike Sattler. Modular Reuse of Ontologies: Theory and Practice. JAIR, 31:273-318, 2008.





New Reasoning Services

- **Conjunctive query** answering
 - Expressive query language for ontologies [1, 2]
$$Q(x, y) \leftarrow C1(x) \wedge C2(y) \wedge R(x, z) \wedge S(z, y)$$
 - Long-standing open problems
 - E.g., decidability of *SHOIQ* conjunctive query answering

[1] Birte Glimm, Ian Horrocks, Carsten Lutz, and Uli Sattler. Conjunctive Query Answering for the Description Logic SHIQ. *JAIR*, 31:157-204, 2008.

[2] Birte Glimm, Ian Horrocks, and Ulrike Sattler. Unions of Conjunctive Queries in SHOQ. In *Proc. of KR 2008*.





Summary

- DLs are a family of **logic based** KR formalisms
 - Useful subsets of **First Order Logic**
 - Basis for **ontology languages** such as **OWL**
- Motivating applications in, e.g., **life sciences** and **semantic web**
- **Reasoning systems** support ontology development & deployment
 - Different **reasoning techniques** for different applications
 - **Robust and scalable** reasoning systems available
- Very **active research area** with many open problems
 - New logics
 - New reasoning tasks
 - New algorithms and implementations
 - ...





Resources

- OWL 2
 - Working group <http://www.w3.org/2007/OWL/wiki/>
 - Language <http://www.w3.org/TR/owl2-overview/>
 - Systems <http://www.w3.org/2007/OWL/wiki/Implementations>
- Tools and Systems
 - <http://www.cs.man.ac.uk/~sattler/reasoners.html>
 - <http://protege.stanford.edu/overview/protege-owl.html>
- Select bibliography
 - F. Baader, I. Horrocks, and U. Sattler. Description Logics. In *Handbook of Knowledge Representation*. Elsevier, 2007.
<http://www.comlab.ox.ac.uk/people/ian.horrocks/Publications/download/2007/BaHS07a.pdf>
 - Ian Horrocks. Ontologies and the semantic web. *Communications of the ACM*, 51(12):58-67, December 2008.
<http://www.comlab.ox.ac.uk/people/ian.horrocks/Publications/download/2008/Horr08a.pdf>