CHAPTER 14: ALLOCATING SCARCE RESOURCES

Multiagent Systems

http://www.csc.liv.ac.uk/~mjw/pubs/imas/

Overview

- Allocation of *scarce* resources amongst a *number of agents* is central to multiagent systems.
- Resource might be:
  - a physical object
  - the right to use land
  - computational resources (processor, memory, . . .)

- If the resource isn't scarce, there is no trouble allocating it.
- If there is no competition for the resource, then there is no trouble allocating it.

- In practice, this means we will be talking about *auctions*.
- These used to be rare (and not so long ago).
- However, auctions have grown massively with the Web/Internet
  - Frictionless commerce
- Now feasible to auction things that weren't previously profitable:
  - eBay
  - Adword auctions

What is an auction?

- Concerned with *traders* and their allocations of:
  - Units of an indivisible *good*; and
  - Money, which is divisible.
- Assume some initial allocation.
- *Exchange* is the free alteration of allocations of goods and money between traders

Limit Price

- Each trader has a value or *limit price* that they place on the good.
- A buyer who exchanges more than their limit price for a good makes a loss.
- A seller who exchanges a good for less than their limit price makes a loss.

- Limit prices clearly have an effect on the behavior of traders.
- There are several models, embodying different assumptions about the nature of the good.
- Three commonly used models:
  - Private value
  - Common value
  - Correlated value
- These are the models you'll find most often adopted in the literature.

Private value

- Good has an value to me that is independent of what it is worth to you.
- Textbook gives the example of John Lennon's last dollar bill.

Common value

- The good has the same value to all of us, but we have differing estimates of what it is.
- Winner's curse

Correlated value

- Our values are related.
- The more you are prepared to pay, the more I should be prepared to pay.

- A *market institution* defines how the exchange takes place.
  - Defines what messages can be exchanged.
  - Defines how the final allocation depends on the messages.
- The change of allocation is market *clearing*.
- Difference between allocations is *net trade*.
  - Component for each trader in the market.
  - Each trader with a non-zero component has a *trade* or *transaction* price.

  - Absolute value of the money component divided by the good component.
- Traders with positive good component are *buyers*
- Traders with negative good component are *sellers*
- *One way traders* are either buyers or sellers but not both.

Yes, but what is an auction?

An *auction* is a market institution in which messages from traders include some price information—this information may be an offer to buy at a given price, in the case of a *bid*, or an offer to sell at a given price, in the case of an *ask*—and which gives priority to higher bids and lower asks.

This definition, as with all this terminology, comes from Dan Friedman.

The zoology of auctions

- We can split auctions into a number of different categories.
- Being good computer scientists, we draw up a taxonomy.
  - This gives us a handle on all the kinds there might be.
  - It suggests parameterization.
  - It can help us to think about implementation.
- This particular classification is a bit zoological, but it is a good place to start.

Single versus multi-dimensional

- Single dimensional auctions
  - The only content of an offer are the price and quantity of some specific type of good.
  - “I’ll bid $200 for those 2 chairs”
- Multi dimensional auctions
  - Offers can relate to many different aspects of many different goods.
  - “I’m prepared to pay $200 for those two red chairs, but $300 if you can deliver them tomorrow.”

Single versus double-sided

- Single-sided markets
  - Either one buyer and many sellers, or one seller and many buyers.
  - The latter is the thing we normally think of as an auction.
- Two-sided markets
  - Many buyers and many sellers.
- Single sided markets with one seller and many buyers are “sell-side” markets.

- Single-sided markets with one buyer and many sellers are “buy-side”.

Open-cry versus sealed-bid

- Open cry
  - Traders announce their offers to all traders
- Sealed bid
  - Only the auctioneer sees the offers.
- Clearly as a bidder in an open-cry auction you have more information.
- In some auction forms you pay for preferential access to information.

Single-unit versus multi-unit

- How many units of the same good are we allowed to bid for?
- Single unit
  - One at a time.
  - Might repeat if many units to be sold.
- Multi-unit
  - Bid both price and quantity.
- “Unit” refers to the indivisible unit that we are selling.
  - Single fish versus box of fish.

First price versus kth price

- Does the winner pay the highest price bid, the second highest price, the kth highest price?

Single item versus multi-item

- Not so much quantity as heterogeneity.
- Single item
  - Just the one indivisible thing that is being auctioned.
- Multi-item
  - Bid for a bundle of goods.
  - "Two red chairs and an orange couch, or a purple beanbag."
  - Valuations for bundles are not linear combinations of the values of the constituents.

Standard auction types

- We will look at the four "standard" auctions:
  - English auction
  - Dutch auction
  - First-price sealed bid auction
  - Vickrey auction
- Also the so-called *Japanese* auction.

English auction

- This is the kind of auction everyone knows.
- Typical example is sell-side.
- Buyers call out bids, bids increase in price.
- In some instances the auctioneer may call out prices with buyers indicating they agree to such a price.
- The seller may set a *reserve price*, the lowest acceptable price.
- Auction ends:
  - at a fixed time (internet auctions); or
  - when there is no more bidding activity.

- The “last man standing” pays their bid.

- Classified in the terms we used above:
  - Single-dimensional
  - Single-sided
  - Open-cry
  - Single unit
  - First-price
  - Single item
- Around 95% of internet auctions are of this kind.
- Classic use is sale of antiques and artwork.

Unlikely tales

The former president of Parke-Benet reports that a dealer attending a sale of eighteenth-century French furniture had arranged to unbutton his overcoat whenever he wished to bid; buttoning the overcoat again would signal that he had ceased bidding. The dealer, coat unbuttoned, was in the midst of bidding for a Louis XVI sofa when he saw someone outside to whom he wished to speak and suddenly left the room. The auctioneer continued to bid for the dealer who, when he returned to the room, found he had become the owner of the sofa at an unexpectedly high price. An argument then followed as to whether an unbuttoned coat not in the auction room is the same as an unbuttoned coat in the auction room.

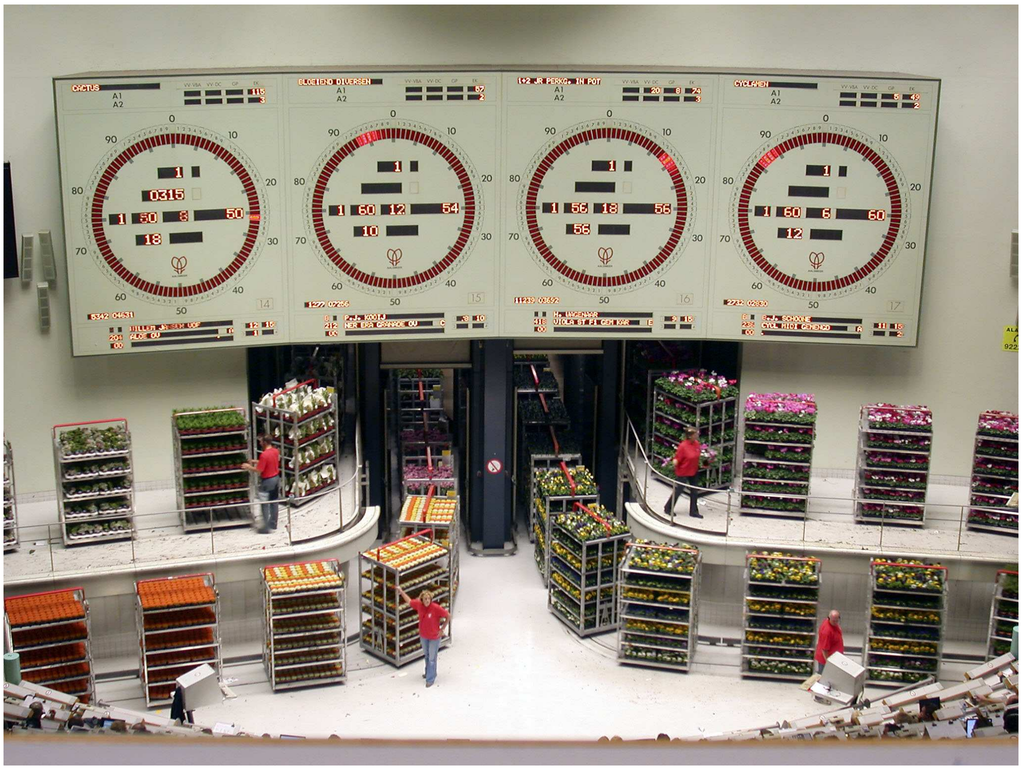
(Cassady, 1969)

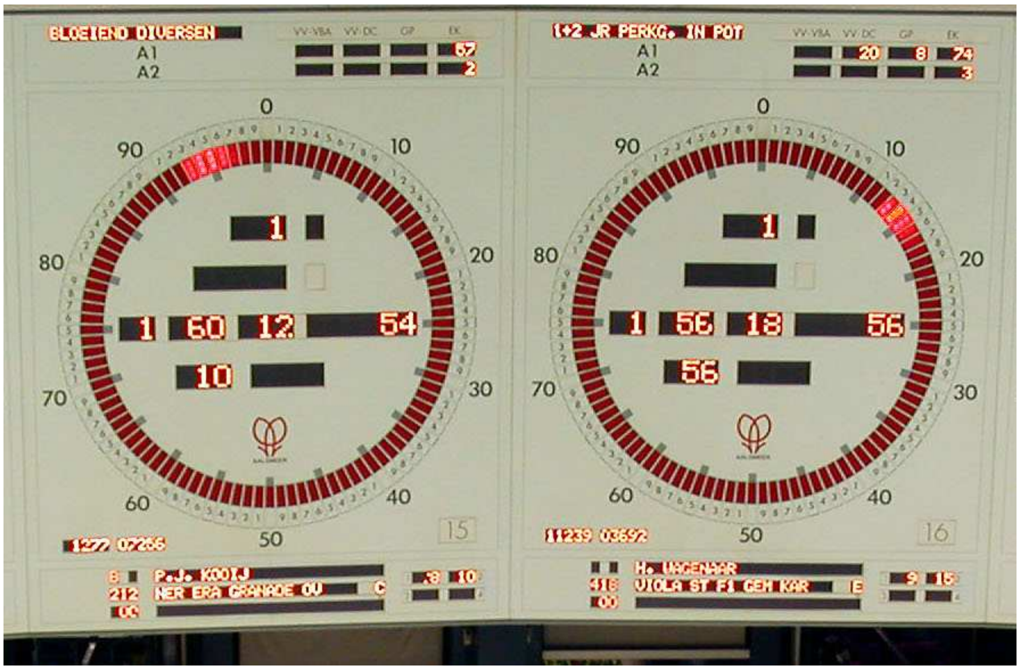
Dutch auction

- Also called a “descending clock” auction
  - Some auctions use a clock to display the prices.
- Starts at a high price, and the auctioneer calls out descending prices.
- One bidder claims the good by indicating the current price is acceptable.

- Ties are broken by restarting the descent from a slightly higher price than the tie occurred at.
- The winner pays the price at which they “stop the clock”.

- Classified in the terms we used above:
  Single-dimensional; Single-sided; Open-cry; Single unit; First-price; Single item
- High volume (since auction proceeds swiftly).
- Often used to sell perishable goods:
  - Flowers in the Netherlands (eg. Aalsmeer)
  - Fish in Spain and Israel.
  - Tobacco in Canada.





- *The Guardian* states that the Aalsmeer auction trades

  19 million flowers and 2 million plants . . . every day.

  April 23rd 2008 (page 18–19)

First-price sealed bid auction

- In an English auction, you get information about how much a good is worth.
- Other people's bids tell you things about the market.
- In a sealed bid auction, none of that happens
  - at most you know the winning price after the auction.
- In the FPSB auction the highest bid wins as always
- As its name suggests, the winner pays that highest price (which is what they bid).

- Classified in the terms we used above:
  - Single-dimensional
  - Single-sided
  - Sealed-bid
  - Single unit
  - First-price
- Governments often use this mechanism to sell treasury bonds.
  - UK still does.
  - US recently changed to SPSB.
- Property can also be sold this way (as in Scotland).

The Amsterdam auction

- Since medieval time, property in the low countries has traditionally been sold using the “Amsterdam” auction.
- Start with an English auction.
- When down to the final two bidders, start a Dutch auction stage.
- Dutch auction starts from twice the final price of the English auction.

Vickrey auctions

- The Vickrey auction is a sealed bid auction.
- The winning bid is the highest bid, but the winning bidder pays the amount of the second highest bid.
- This sounds odd, but it is actually a very smart design.
- It is in the bidders’ interest to bid their true value.
  - *incentive compatible* in the usual terminology.
- However, it is not a panacea, as the New Zealand government found out.

- Again, classified as above, it is:
  - Single-dimensional
  - Single-sided
  - Sealed-bid
  - Single unit
  - Second-price

Why does the Vickrey auction work?

- Suppose you bid *more* than your valuation.
  - You may win the good.
  - If you do, you may end up paying more than you think the good is worth.
  - Not so smart.

- Suppose you bid *less* than your valuation.
  - You stand less chance of winning the good.
  - However, even if you do win it, you will end up paying the same.
  - Not so smart.

- So: *there is no point in bidding above or below your valuation*.
- Of course, this really assumes there are a large number of bidders (see the New Zealand case).

Japanese fish auction

- The auction form used to sell fish in Tokyo is different:

  > [The] distinctive aspect [of this auction form] is that all bids are made by prospective buyers at the same time, or approximately the same time, using individual hand signs for each monetary unit. . . . The bidding starts as soon as the auctioneer gives the signal, and the highest bidder, as determined by the auctioneer, is awarded the lot.

- This is thus *simultaneous bidding* and rather like an FPSB auction.

- Ties are “not uncommon[ly]” broken by playing Jan Ken Pon (or ‘paper, rock, scissors’).

Combinatorial Auctions

- Auctions for bundles of goods.
- A good example of bundles of good are spectrum licences.
- For the 1.7 to 1.72 GHz band for Brooklyn to be useful, you need a license for Manhattan, Queens, Staten Island.
- Most valuable are the licenses for the same bandwidth.
- But a different bandwidth licence is more valuable than no license

- (The FCC spectrum auctions, however, did not use a combinatorial auction mechanism)

- Let $\mathcal{Z} = \{z_1, \ldots, z_m\}$ be a set of items to be auctioned.
- We gave the usual set of agents $Ag = \{1, \ldots, n\}$, and we capture preferences of agent i with the *valuation* function:

$$v_i : 2^{\mathcal{Z}} \mapsto \mathbb{R}$$

  meaning that for every possible bundle of goods $Z \subseteq \mathcal{Z}$, $v_i(Z)$ says how much Z is worth to i.
- If $v_i(\emptyset) = 0$, then we say that the valuation function for i is normalised.

- Another useful idea is *free disposal*:

$$Z_1 \subseteq Z_2 \text{ implies } v_i(Z_1) \leq v_i(Z_2)$$

- In other words, an agent is never worse off having more stuff.

- We already mentioned the idea of an allocation.
- Formally an allocation is a list of sets $Z_1, \ldots Z_n$, one for each agent Ag_i with the stipulation that:

$$Z_i \subseteq \mathcal{Z}$$

  and for all $i, j \in Ag$ such that $i \neq j$, we have $Z_i \cap Z_j = \emptyset$.
- Thus no good is allocated to more than one agent.
- The set of all allocations of $\mathcal{Z}$ to agents Ag is:

$$alloc(\mathcal{Z}, Ag)$$

- If we design the auction, we get to say how the allocation is determined.
- How should this be?
- One natural way is to *maximize social welfare*.
  - Sum of the utilities of all the agents.
- Define a social welfare function:

$$sw(Z_1, \ldots, Z_n, v_1, \ldots, v_n) = \sum_{i=1}^{n} v_i(Z_i)$$

- Given this, we can define a *combinatorial auction*.
- Given a set of goods $\mathcal{Z}$ and a collection of valuation functions $v_1, \ldots, v_n$, one for each agent $i \in Ag$, the goal is to find an allocation

$$Z_1^*, \ldots, Z_n^*$$

that maximizes sw, in other words

$$Z_1^*, \ldots, Z_n^* = \arg \max_{(Z_1, \ldots, Z_n) \in alloc(\mathcal{Z}, Ag)} sw(Z_1, \ldots, Z_n, v_1, \ldots, v_n)$$

- Figuring this out is *winner determination*.

- How do we do this?
- Well, we could get every agent i to declare their valuation $\hat{v}_i$
  - The hat denotes that this is what the agent *says*, not what it necessarily is.
  - The agent may lie!
- Then we just look at all the possible allocations and figure out what the best one is.

- One problem here is *representation*, valuations are exponential:

$$v_i : 2^{\mathcal{Z}} \mapsto \mathbb{R}$$

  - A naive representation is impractical.
  - In a bandwidth auction with 1122 licenses we would have to specify 2^{1122} values for each bidder.
- Searching through them is computationally intractable.

Bidding languages

- Rather than exhaustive evaluations, allow bidders to construct valuations from the bits they want to mention.
- Atomic bids (Z, p) where $Z \subseteq \mathcal{Z}$.
- A bundle Z' *satisfies* a bid (Z, p) if $Z \subseteq Z'$.
- In other words a bundle satisifes a bid if it contains at least the things in the bid.

- Atomic bids define valuations

$$v_\beta(Z') = \begin{cases} p & \text{if } Z' \text{ satisfies } (Z, p) \\ 0 & \text{otherwise} \end{cases}$$

- Atomic bids alone don't allow us to construct very interesting valuations.

- To construct more complex valuations, atomic bids can be combined into more complex bids.
- One approach is XOR bids

$$B_i = (\{a, b\}, 3) \; XOR \; (\{c, d\}, 5)$$

- XOR because we will pay for *at most one*.
- We read the bid to mean:

  I would pay 3 for a bundle that contains a and b but not c and d. I will pay 5 for a bundle that contains c and d but not a and b, and I will pay 5 for a bundle that contains a, b, c and d.

- From this we can construct a valuation.

- Thus:

$$\begin{aligned} v_{\beta_1}(\{a\}) &= 0 \\ v_{\beta_1}(\{b\}) &= 0 \\ v_{\beta_1}(\{a, b\}) &= 3 \\ v_{\beta_1}(\{c, d\}) &= 5 \\ v_{\beta_1}(\{a, b, c, d\}) &= 5 \end{aligned}$$

- More formally, a bid like this:

$$\beta = (Z_1, p_1) XOR \ldots XOR (Z_k, p_k)$$

defines a valuation v_β like so:

$$v_\beta(Z') = \begin{cases} 0 & \text{if } Z' \text{ doesn't satisfy any } (Z_i, p_i) \\ \max\{p_i \mid Z_i \subseteq Z'\} & \text{otherwise} \end{cases}$$

- XOR bids are *fully expressive*, that is they can express any valuation function over a set of goods.
- To do that, we may need an exponentially large number of atomic bids.
- However, the valuation of a bundle can be computed in polynomial time.

Winner Determination

- The basic problem is intractable.
- But this is a worst case result, so it may be possible to develop approaches thatare optimal and run well in many cases.
- Can also forget optimality and either:
  - use heuristics; or
  - look for approximation algorithms.
- Common approach: code the problem as an integer linear program and use a standard solver – often works in practice.

The VCG Mechanism

- In general we don't know whether the $\hat{v}_i$ are true valuations.
- Life would be easier if they were!
  - Well, can we make them true valuations?
- Yes, in a generalization of the Vickrey auction.
  - Vickrey/Clarke/Groves Mechanism
- Mechanism is *incentive compatible*: telling the truth is a dominant strategy.

- Need some more notation.
- Indifferent valuation function:

$$v^0(Z) = 0$$

for all Z.

- sw_{-i} is the social welfare function without i:

$$sw_{-i}(Z_1, \ldots, Z_n, v_1, \ldots, v_n) = \sum_{j \in Ag, j \neq i} v_j(Z_j)$$

- And we can then define the VCG mechanism.

1. Every agent simultaneously declares a valuation $\hat{v}_i$.

2. The mechanism computes:

$$Z_1^*, \ldots, Z_n^* = \arg \max_{(Z_1,\ldots,Z_n) \in alloc(\mathcal{Z}, Ag)} sw(Z_1, \ldots, Z_n, \hat{v}_1, \ldots, \hat{v}_i, \ldots, \hat{v}_n)$$

and the allocation $Z_1^*, \ldots, Z_n^*$ is chosen.

3. The mechanism also computes, for each agent i:

$$Z_1', \ldots, Z_n' = argmax_{(Z_1,\ldots,Z_n) \in alloc(\mathcal{Z}, Ag)} sw(Z_1, \ldots, Z_n, \hat{v}_1, \ldots, v^0, \ldots, \hat{v}_n)$$

the allocation that maximises social welfare were that agent to have declared v^0 to be its valuation.

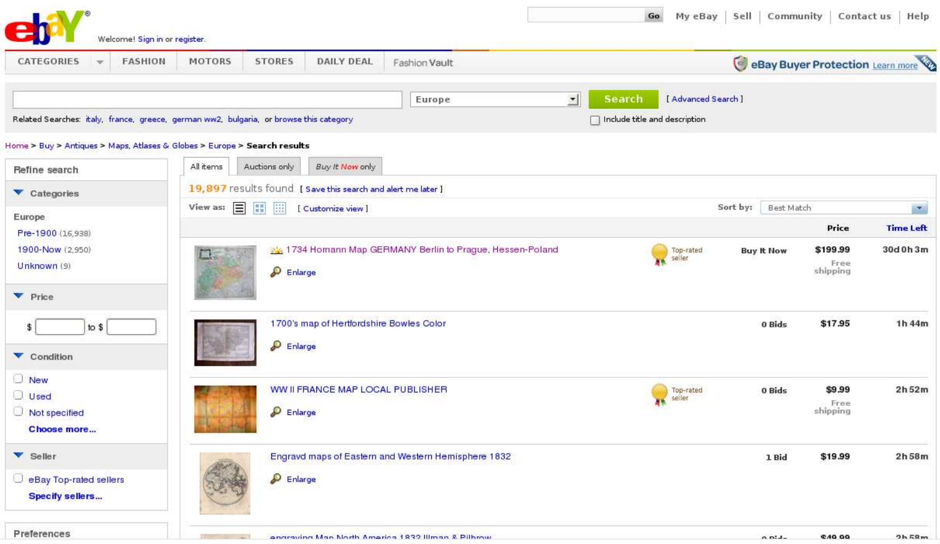
4. Every agent i pays p_i, where:

$$\begin{aligned} p \;=\; & sw_{-i}(Z'_1, \ldots, Z'_n, \hat{v}_1, \ldots, v^0, \ldots, v_n) \\ & - sw_{-i}(Z^*_1, \ldots, Z^*_n, \hat{v}_1, \ldots, \hat{v}_i, \ldots, v_n) \end{aligned}$$

- On other words, each agent pays out the cost, to other agents, of it having participated in the auction.
- It is incentive compatible for exactly the same reason as the Vickrey auction was before.
- If you bid more than your valuation and win, well you end up paying back what the good is worth to everyone else, which is more than it is worth to you.
- If you shade your bid, you reduce your chance to win, but even if you win you are still paying what everyone else thinks it is worth so you don't save money by reducing yoru chance to win.

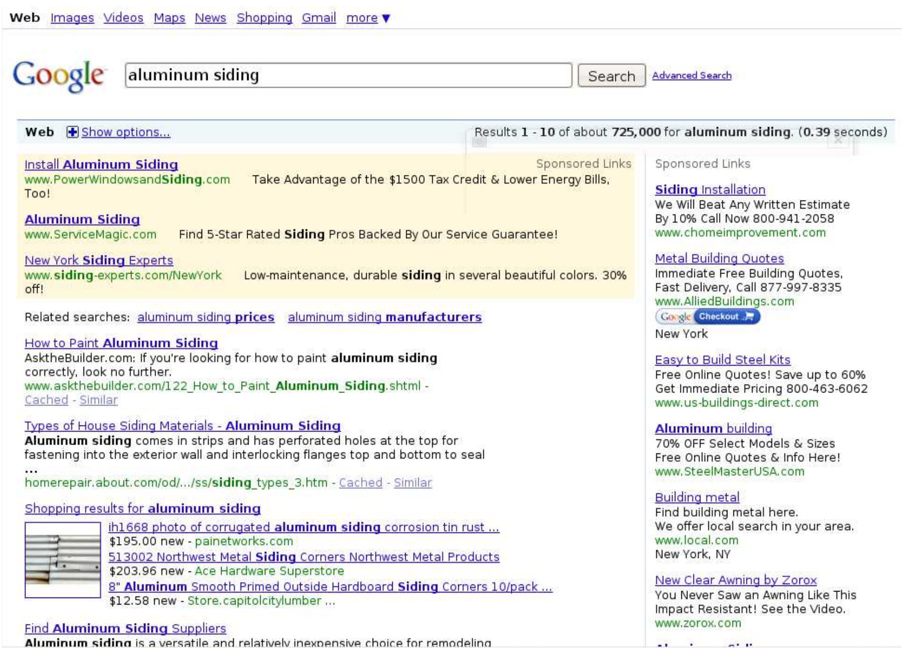
- So we get a dominant strategy for each agent that guarantees to maximise social welfare.

eBay



- eBay runs a variation of the English auction.
- Vulnerable to *sniping*.
- To counter this, eBay offers a automated bidding agent.
  – Reduces the auction to a FPSB.
- Many companies offer sniping services.
- BTW, there is an easy fix to sniping, but eBay chose not to use it.
  – Activity rule

Adword auctions



- To decide which ads get shown in which position for which searches, an adword auction is run.
- This is run *in real time*.
- (Though clearly bids are placed beforehand.)
- Auction is a variation on the Vickrey auction.
- 85% of Google's revenue ($4.1 billion) in 2005 came from these auctions.
- Very active area of research.
  - Not clear what the best auction mechanism is for this application.
  - Not clear what the best way to bid is.

Summary

- Allocating scarce resources comes down to auctions.
- We looked at a range of different simple auction mechanisms.
  - English auction
  - Dutch auction
  - First price sealed bid
  - Vickrey auction
- The we looked at the popular field of combinatorial auctions.

- We discussed some of the problems in implementing combinatorial auctions.
- And we talked about the Vickrey/Clarke/Groves mechanism, a rare ray of sunshine on the problems of multiagent interaction.