CHAPTER 11: MULTIAGENT INTERACTIONS

An Introduction to Multiagent Systems

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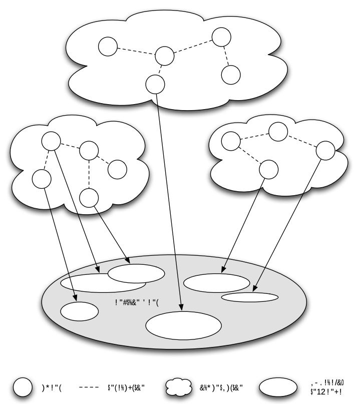
1 What are Multiagent Systems?



Thus a multiagent system contains a number of agents …

- … which interact through communication …
- … are able to act in an environment …
- … have different "spheres of influence" (which may coincide)…
- … will be linked by other (organisational) relationships.

2 Utilities and Preferences

- Assume we have just two agents: $Ag = \{i, j\}$.
- Agents are assumed to be *self-interested*: they *have preferences over how the environment is*.
- Assume $\Omega = \{\omega_1, \omega_2, \ldots\}$ is the set of "outcomes" that agents have preferences over.
- We capture preferences by *utility functions*:

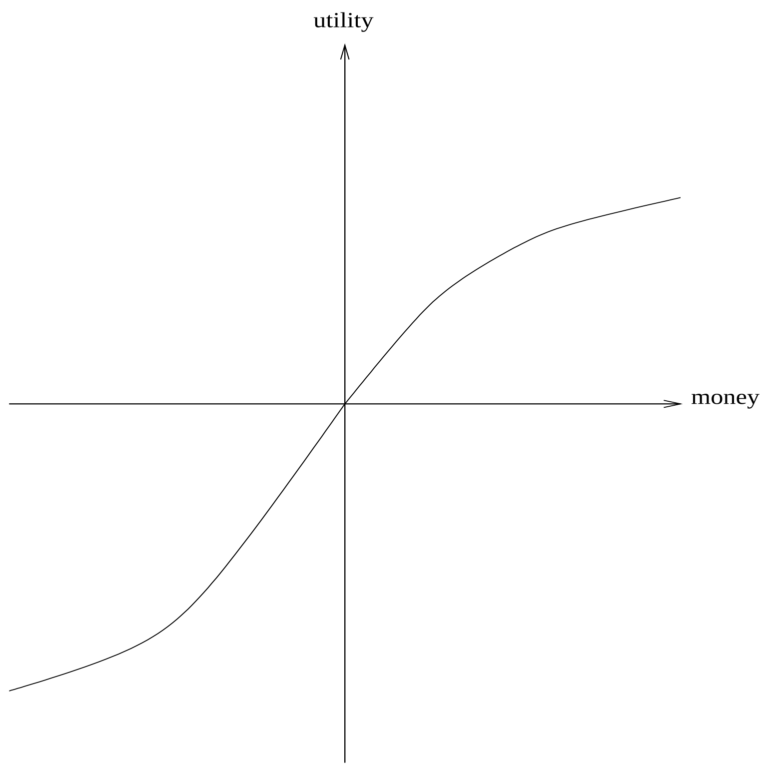
$$u_i : \Omega \to \mathbb{R}$$
$$u_j : \Omega \to \mathbb{R}$$

- Utility functions lead to *preference orderings* over outcomes:

$$\omega \succeq_i \omega' \;\text{ means }\; u_i(\omega) \geq u_i(\omega')$$
$$\omega \succ_i \omega' \;\text{ means }\; u_i(\omega) > u_i(\omega')$$

What is Utility?

- Utility is *not* money (but it is a useful analogy).
- Typical relationship between utility & money:


utility
money

3 Multiagent Encounters

- We need a model of the environment in which these agents will act. . .
  - agents simultaneously choose an action to perform, and as a result of the actions they select, an outcome in Ω will result;
  - the *actual* outcome depends on the *combination* of actions;
  - assume each agent has just two possible actions that it can perform C (“cooperate”) and “D” (“defect”).

- Environment behaviour given by *state transformer function*:

$$\tau : \underbrace{Ac}_{\text{agent } i\text{'s action}} \times \underbrace{Ac}_{\text{agent } j\text{'s action}} \rightarrow \Omega$$

- Here is a state transformer function:

  $$\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2 \quad \tau(C,D) = \omega_3 \quad \tau(C,C) = \omega_4$$

  (This environment is sensitive to actions of both agents.)

- Here is another:

  $$\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_1 \quad \tau(C,D) = \omega_1 \quad \tau(C,C) = \omega_1$$

  (Neither agent has any influence in this environment.)

- And here is another:

  $$\tau(D,D) = \omega_1 \quad \tau(D,C) = \omega_2 \quad \tau(C,D) = \omega_1 \quad \tau(C,C) = \omega_2$$

  (This environment is controlled by j.)

Rational Action

- Suppose we have the case where *both* agents can influence the outcome, and they have utility functions as follows:

$$\begin{array}{llll} u_i(\omega_1) = 1 & u_i(\omega_2) = 1 & u_i(\omega_3) = 4 & u_i(\omega_4) = 4 \\ u_j(\omega_1) = 1 & u_j(\omega_2) = 4 & u_j(\omega_3) = 1 & u_j(\omega_4) = 4 \end{array}$$

- With a bit of abuse of notation:

$$\begin{array}{llll} u_i(D,D) = 1 & u_i(D,C) = 1 & u_i(C,D) = 4 & u_i(C,C) = 4 \\ u_j(D,D) = 1 & u_j(D,C) = 4 & u_j(C,D) = 1 & u_j(C,C) = 4 \end{array}$$

- Then agent i's preferences are:

$$C,C \succeq_i C,D \quad \succ_i \quad D,C \succeq_i D,D$$

- "C" is the *rational choice* for i.

  (Because i prefers all outcomes that arise through C over all outcomes that arise through D.)

Payoff Matrices

- We can characterise the previous scenario in a *payoff matrix*

		i	
		defect	coop
j	defect	1 1	4 1
	coop	1 4	4 4

- Agent i is the *column player*.
- Agent j is the *row player*.

Solution Concepts

- How will a rational agent will behave in any given scenario?
- Answered in *solution concepts*:
  - dominant strategy;
  - Nash equilibrium strategy;
  - Pareto optimal strategies;
  - strategies that maximise social welfare.

Dominant Strategies

- We will say that a strategy s_i is *dominant* for player i if no matter what strategy s_j agent j chooses, i will do at least as well playing s_i as it would doing anything else.
- Unfortunately, there isn't always a dominant strategy.

(Pure Strategy) Nash Equilibrium

- In general, we will say that two strategies s_1 and s_2 are in Nash equilibrium if:
  1. under the assumption that agent i plays s_1, agent j can do no better than play s_2; and
  2. under the assumption that agent j plays s_2, agent i can do no better than play s_1.
- *Neither agent has any incentive to deviate from a Nash equilibrium.*
- Unfortunately:

1. *Not every interaction scenario has a Nash equilibrium.*
2. *Some interaction scenarios have more than one Nash equilibrium.*

Matching Pennies

Players i and j simultaneously choose the face of a coin, either “heads” or “tails”.
If they show the same face, then i wins, while if they show different faces, then j wins.

Matching Pennies: The Payoff Matrix

	i heads	i tails
j heads	1 -1	-1 1
j tails	-1 1	1 -1

Mixed Strategies for Matching Pennies

- NO pair of strategies forms a pure strategy NE: whatever pair of strategies is chosen, somebody will wish they had done something else.
- The solution is to allow *mixed strategies*:
  - play "heads" with probability 0.5
  - play "tails" with probability 0.5.
- This is a NE strategy.

Mixed Strategies

- A mixed strategy has the form
  - play α_1 with probability p_1
  - play α_2 with probability p_2
  - ...
  - play α_k with probability p_k.

  such that $p_1 + p_2 + \cdots + p_k = 1$.
- Nash proved that *every finite game has a Nash equilibrium in mixed strategies*.

Nash's Theorem

- Nash proved that *every finite game has a Nash equilibrium in mixed strategies*. (Unlike the case for *pure strategies*.)
- So this result overcomes the lack of solutions; but there still may be more than one Nash equilibrium. . .

Pareto Optimality

- An outcome is said to be *Pareto optimal* (or *Pareto efficient*) if there is no other outcome that makes one agent *better off* without making another agent *worse off*.
- If an outcome is Pareto optimal, then at least one agent will be reluctant to move away from it (because this agent will be worse off).

- If an outcome ω is *not* Pareto optimal, then there is another outcome ω' that makes *everyone* as happy, if not happier, than ω.

  "Reasonable" agents would agree to move to ω' in this case. (Even if I don't directly benefit from ω', you can benefit without me suffering.)

Social Welfare

- The social welfare of an outcome ω is the sum of the utilities that each agent gets from ω:

$$\sum_{i \in Ag} u_i(\omega)$$

- Think of it as the "total amount of money in the system".
- As a solution concept, may be appropriate when the whole system (all agents) has a single owner (then overall benefit of the system is important, not individuals).

Competitive and Zero-Sum Interactions

- Where preferences of agents are diametrically opposed we have *strictly competitive* scenarios.
- Zero-sum encounters are those where utilities sum to zero:

$$u_i(\omega) + u_j(\omega) = 0 \quad \text{for all } \omega \in \Omega.$$

- Zero sum encounters are bad news: for me to get +ve utility *you have to get negative utility*! The best outcome for me is the *worst* for you!

- Zero sum encounters in real life are very rare . . . but people frequently act as if they were in a zero sum game.

4 The Prisoner's Dilemma

Two men are collectively charged with a crime and held in separate cells, with no way of meeting or communicating.

They are told that:

- if one confesses and the other does not, the confessor will be freed, and the other will be jailed for three years;
- if both confess, then each will be jailed for two years.

Both prisoners know that if neither confesses, then they will each be jailed for one year.

- Payoff matrix for prisoner's dilemma:

		i	
		defect	coop
j	defect	2 2	1 4
	coop	4 1	3 3

- Top left: If both defect, then both get punishment for mutual defection.
- Top right: If i cooperates and j defects, i gets sucker's payoff of 1, while j gets 4.

- Bottom left: If j cooperates and i defects, j gets sucker's payoff of 1, while i gets 4.
- Bottom right: Reward for mutual cooperation.

What Should You Do?

- The *individual rational* action is *defect*.

  This guarantees a payoff of no worse than 2, whereas cooperating guarantees a payoff of at most 1.
- So defection is the best response to all possible strategies: both agents defect, and get payoff = 2.
- But *intuition* says this is *not* the best outcome:

  Surely they should both cooperate and each get payoff of 3!

Solution Concepts

- D is a dominant strategy.
- (D, D) is the only Nash equilibrium.
- All outcomes *except* (D, D) are Pareto optimal.
- (C, C) maximises social welfare.

- This apparent paradox is *the fundamental problem of multi-agent interactions*.

  It appears to imply that *cooperation will not occur in societies of self-interested agents*.
- Real world examples:
  - nuclear arms reduction ("why don't I keep mine. . . ")
  - free rider systems — public transport;
  - in the UK — television licenses.
- The prisoner's dilemma is *ubiquitous*.
- Can we recover cooperation?

Arguments for Recovering Cooperation

- Conclusions that some have drawn from this analysis:
  - the game theory notion of rational action is wrong!
  - somehow the dilemma is being formulated wrongly
- Arguments to recover cooperation:
  - We are not all machiavelli!
  - The other prisoner is my twin!
  - Program equilibria and mediators
  - The shadow of the future...

4.1 Program Equilibria

- The strategy you *really* want to play in the prisoner's dilemma is:

  I'll cooperate if he will

  .

- Program equilibria provide one way of enabling this.
- Each agent submits a *program strategy* to a *mediator* which *jointly executes* the strategies.

  Crucially, strategies can be *conditioned on the strategies of the others*.

4.2 Program Equilibria

- Consider the following program:

```
IF HisProgram == ThisProgram THEN
  DO(C);
ELSE
  DO(D);
END-IF.
```

  Here == is *textual comparison*.

- The best response to this program is *to submit the same program*, giving an outcome of (C, C)!

- You *can't* get the sucker's payoff by submitting this program.

4.3 The Iterated Prisoner's Dilemma

- One answer: *play the game more than once*.

  If you know you will be meeting your opponent again, then the incentive to defect appears to evaporate.

- *Cooperation is the rational choice in the infinititely repeated prisoner's dilemma.*

  (Hurrah!)

4.4 Backwards Induction

- But. . . suppose you both know that you will play the game exactly n times.

  On round $n-1$, you have an incentive to defect, to gain that extra bit of payoff. . .

  But this makes round $n-2$ the last "real", and so you have an incentive to defect there, too.

  This is the *backwards induction* problem.
- Playing the prisoner's dilemma with a fixed, finite, pre-determined, commonly known number of rounds, defection is the best strategy.

4.5 Axelrod's Tournament

- Suppose you play iterated prisoner's dilemma against a *range* of opponents . . .

  What strategy should you choose, so as to maximise your overall payoff?
- Axelrod (1984) investigated this problem, with a computer tournament for programs playing the prisoner's dilemma.

Strategies in Axelrod's Tournament

- ALLD:
  "Always defect" — the *hawk* strategy;
- TIT-FOR-TAT:
  1. On round $u = 0$, cooperate.
  2. On round $u > 0$, do what your opponent did on round $u - 1$.
- TESTER:
  On 1st round, defect. If the opponent retaliated, then play TIT-FOR-TAT. Otherwise intersperse cooperation & defection.

- JOSS:

  As TIT-FOR-TAT, except periodically defect.

Recipes for Success in Axelrod's Tournament

Axelrod suggests the following rules for succeeding in his tournament:

- *Don't be envious*:

  Don't play as if it were zero sum!

- *Be nice*:

  Start by cooperating, and reciprocate cooperation.

- *Retaliate appropriately*:

  Always punish defection immediately, but use "measured" force — don't overdo it.

- *Don't hold grudges*:

  Always reciprocate cooperation immediately.

5 Game of Chicken

- Consider another type of encounter — the *game of chicken*:

		i	
		defect	coop
j	defect	1 (top right), 1 (bottom left)	2 (top right), 4 (bottom left)
	coop	4 (top right), 2 (bottom left)	3 (top right), 3 (bottom left)

(Think of James Dean in *Rebel without a Cause*: swerving = coop, driving straight = defect.)

- Difference to prisoner's dilemma:

Mutual defection is most feared outcome.

(Whereas sucker's payoff is most feared in prisoner's dilemma.)

Solution Concepts

- There is no dominant strategy (in our sense).
- Strategy pairs (C, D)) and (D, C)) are Nash equilibriums.
- All outcomes except (D, D) are Pareto optimal.
- All outcomes except (D, D) maximise social welfare.

6 Other Symmetric 2 x 2 Games

- Given the 4 possible outcomes of (symmetric) cooperate/defect games, there are 24 possible orderings on outcomes.
  - $CC \succ_i CD \succ_i DC \succ_i DD$
    Cooperation dominates.
  - $DC \succ_i DD \succ_i CC \succ_i CD$
    Deadlock. You will always do best by defecting.
  - $DC \succ_i CC \succ_i DD \succ_i CD$
    Prisoner's dilemma.
  - $DC \succ_i CC \succ_i CD \succ_i DD$
    Chicken.

– $CC \succ_i DC \succ_i DD \succ_i CD$

Stag hunt.