## Lecture 2 — Propositional Logic (1)

1. Which, (if any), of the following sentences is a proposition? In each case, justify your answer.

   (a) Twice two is four.

   (b) The square root of two is not a rational number.

   (c) $A \subset B$

   (d) $5 > 6$

   (e) Pigs can fly.

   (f) The *Stone Roses* album is a classic.

   (g) *Guns & Roses* suck.

   (h) Bjork is overrated.

   (i) Please write a specimen of your signature in the space provided.

   (j) Would you believe you're standing in a place where Madonna once stood?

   (k) That's true.

   (l) I promise not to peep.

   (m) Adultery is wicked.

2. Identify the propositions in each of the following examples, and rewrite them as propositional logic formulae. Draw up truth tables for each of the examples.

   (a) If the reactor is on, then the warning system should be on.

   (b) If the temperature is high, and the reactor is on, then the red light should be on, and the green light should be off.

   (c) If the temperatire is low, and the reactor is on, then the green light should be on and the red light should be off.

   (d) If valve-5 is open and valve-6 is not open, then if the air vent is on, then the cooling system should be on.

3. Symbolise the following propositions:

   (a) Charlie is a chimp and Charlie likes bananas.

   (b) The Chesire cat vanished and its grin remained.

   (c) $x \in A$ and $x \in B$.

   (d) I'll party tonight or I'll stay in and watch television.

4. Using the following propositions:

| | |
|---|---|
| $p$ | Its is raining |
| $q$ | I have an umbrella |
| $r$ | I get wet |

Formulate the following expressions in words:

(a) $(p \wedge q)$

(b) $(p \wedge \neg q) \wedge r$

(c) $\neg p \wedge \neg r$

(d) $p \wedge (q \vee r)$

(e) $\neg p \vee r$

(f) $\neg(p \vee r)$

5. Construct truth tables for the expressions in the previous question.

6. Determine, for each of the following propositional logic formulae, whether it is:

- a tautology;
- consistent;
- inconsistent.

(a) $p \vee \neg p$

(b) $(p \Rightarrow q) \Rightarrow ((p \Rightarrow r) \Rightarrow (p \Rightarrow r))$

(c) $(p \wedge q) \wedge \neg(p \vee q)$

(d) $p \vee q \vee \neg r$

(e) $(p \wedge q) \Rightarrow r$

(f) $p \Rightarrow (q \Rightarrow p)$

(g) $p \Rightarrow (p \Rightarrow q)$

(h) $p \Leftrightarrow p$

(i) $(p \Leftrightarrow q) \Rightarrow (p \Rightarrow q)$

(j) $(p \Rightarrow q) \Leftrightarrow ((\neg p) \vee q)$

(k) $(p \vee q) \Leftrightarrow \neg(\neg p \wedge \neg q)$

(l) $(p \Rightarrow q) \Rightarrow (\neg q \Rightarrow \neg p)$

(m) $(p \wedge (q \vee p) \Leftrightarrow \neg p$

7. Using the following propositions:

| | |
|---|---|
| $p$ | Logic is easy |
| $q$ | There is a logic question |
| $r$ | I pass the exam |

symbolise the following statements:

(a) If logic is easy then I pass the exam.

(b) If I pass the exam then logic is easy.

(c) I fail the exam if logic is difficult.

(d) If there is no logic question, then I pass the exam.

(e) Logic is difficult. I pass the exam if there is no logic question.

(f) Either logic is easy, and I pass the exam, or logic is hard, and I fail.

(g) If logic is easy, then if there is a logic question, then I pass the exam.

(h) If logic is difficult then I pass the exam if there is no logic question.

(i) If I fail the exam, then there is a logic question and logic is difficult.