Lecture 3 — Sets

1. Which of the following sets are equal:

   (a) $\{r, t, s\}$

   (b) $\{s, t, r, s\}$

   (c) $\{t, s, t, r\}$

   (d) $\{s, r, s, t\}$

2. Consider the following sets: $\emptyset$, $A == \{1\}$, $B == \{1, 3\}$, $C == \{1, 5, 9\}$, $D == \{1, 2, 3, 4, 5\}$, $E == \{1, 3, 5, 7, 9\}$, $U = \{1, 2, \ldots, 8, 9\}$.

   Now insert the correct symbol (either $\subseteq$ or $\nsubseteq$) between each of the pairs of set, to make the expression TRUE.

   (a) $\emptyset, A$

   (b) A, B

   (c) B, C

   (d) B, E

   (e) C, D

   (f) C, E

   (g) D, E

   (h) D, U

3. For the following problems, assume $A == \{1, \ldots, 5\}$, $B == \{4, \ldots, 7\}$, $C == \{5, \ldots, 9\}$, $D == \{1, 3, 5, 7, 9\}$, $E == \{2, 4, 6, 8\}$, $F == \{1, 5, 9\}$

   Find:

   (a) $A \cup B$

   (b) $A \cap B$

   (c) $B \cup D$

   (d) $B \cap D$

   (e) $A \cup C$

   (f) $A \cap C$

   (g) $D \cup E$

   (h) $D \cap E$

   (i) $E \cup E$

   (j) $E \cap E$

(k) $D \cup F$
(l) $D \cap F$
(m) $A \setminus B$
(n) $B \setminus A$
(o) $D \setminus E$
(p) $F \setminus D$
(q) $A \cap (B \cup E)$
(r) $(A \cap D) \setminus B$
(s) $(B \cap F) \cup (C \cap E)$

4. In the following expressions, replace the question mark so as to make a true statement about sets.

    (a) $A \cup A =?$
    (b) $A \cap \emptyset =?$
    (c) $(A \subseteq B) \Leftrightarrow (A \setminus B =?)$
    (d) $A \cap A =?$
    (e) $A \cup \emptyset =?$
    (f) $(A \subseteq B) \wedge (B \subseteq C) \Rightarrow (A?C)$
    (g) $\emptyset ? A$

5. List the elements of each of the following sets:

    (a) $\{n \mid (n \in \mathbb{N}) \wedge (n > 6) \wedge (n > 10)\}$
    (b) $\{n \mid (n \in \mathbb{N}) \wedge (n > 7) \wedge (n \leq 8)\}$

6. For each of the sets in Question 2, define a set enumeration expression containing those members. So for the set A, this expression will be:

$$\{n \mid (n \in \mathbb{N}) \wedge (n > 0) \wedge (n < 2)\}$$

7. For each of the sets in Question 2, write down the *powerset* of that set.

8. (HARD) Lewis Carroll is said to have made the following statements:

    - A dishonourable man is never perfect.
    - An honourable man never tells lies.
    - A man is not perfect unless he is always tactful.
    - Every tactful man tells occasional lies.

    Express each of these statements in set-theoretic terms. What can you deduce about the set of perfect men?