Lecture 4 — First-Order (Predicate) Logic

1. Convert the following sentences to predicate logic form:

   (a) A transaction is invalid if its amount is greater that £500.

   (b) A transaction is valid if its code is greater than 1000 and the transaction amount lies between £10 and £500.

   (c) All the customers are in credit.

   (d) Every prime number other than 2 is odd.

   (e) All suppliers of part p are based in Manchester.

   (f) Any request for personal data must be handled within 14 days.

   (g) Every cloud has a silver lining.

   (h) All the bells in heaven shall ring.

   (i) Each student must hand in homework.

   (j) Nobody knows the trouble I seen.

   (k) Roses are red.

   (l) At least one command line will be active.

   (m) At least one VDU will be connected to the peripheral file-store.

   (n) If at least one command line is down, then the system i in an error state.

   (o) The sum of any two odd integers is even.

   (p) Everybody hates grunge music.

   (q) Everybody hates all grunge music except that by Nirvana.

   (r) x is an male ancestor of y means that either (i) x is the father of y or (ii) there is somebody else – call them z — such that x is the father of z and z is an ancestor of y.

2. Which of the following statements is TRUE; argue informally in each case to justify your answer.

   (a) $\exists i : \mathbb{N} \bullet i = i$

   (b) $\forall i : \mathbb{N} \bullet i = i$

   (c) $\exists i : \mathbb{N} \bullet i \neq i$

   (d) $\forall i, j : \mathbb{N} \bullet i \neq j$

   (e) $\forall x : \mathbb{N} \bullet \exists y : \mathbb{N} \bullet (y < x)$

   (f) $\forall x : \mathbf{Z} \bullet \exists y : \mathbf{Z} \bullet (y < x)$

   (g) $\forall i, j : \mathbb{N} \bullet (i \leq j) \Rightarrow \exists k : \mathbb{N} \bullet ((i + k) = j)$

3. Let

$Days$	be the set of days
$R(X)$	mean that x is rainy
$S(y)$	mean that x is sunny

Symbolise each of the following in two different ways:

(a) All days are sunny.

(b) Some days are not sunny.

(c) Every day that is not sunny is rainy.

(d) No day is both sunny and rainy.

(e) No day is sunny.

4. Express the fact that there is not a largest integer using predicate logic.