Lecture 5 — Functions

1. Let

$$\begin{aligned} A &== \{a, b, c\} \\ B &== \{x, y, z\} \end{aligned}$$

and

$$\begin{aligned} f_1 &== \{a \mapsto y, c \mapsto x\} \\ f_2 &== \{a \mapsto y, b \mapsto z, c \mapsto x, c \mapsto z\} \\ f_3 &== \{a \mapsto y, b \mapsto y, c \mapsto z\} \\ f_4 &== \{a \mapsto z, b \mapsto y, c \mapsto x\}. \end{aligned}$$

(a) For each $f_i \in \{f_1, \ldots, f_4\}$, say whether f_i has the property of being:

i. a partial function from A to B;

ii. a total funtion from A to B;

iii. an injection from A to B;

iv. a surjection from A to B;

v. a bijection from A to B.

If in any case your answer is negative, then justify it.

(b) For each f_i that *is* a function, give

i. $\operatorname{dom} f_i$

ii. $\operatorname{ran} f_i$.

(c) For each of the following, say whether it is a legal expression, and if it is legal, evaluate it.

i. $f_1 \cup f_2$

ii. $(f_1 \cup f_4)(a)$

iii. $f_1 \oplus \{a \mapsto z\}$

iv. $(f_1 \oplus \{a \mapsto z\}) \cup \{b \mapsto y\}$

v. $((f_1 \oplus \{a \mapsto z\}) \setminus \{a \mapsto z\}$

2. The set of partial functions from type T_1 to type T_2 is given by the expression

$$T_1 \nrightarrow T_2.$$

We can use this expression in a set comprehension which gives the set of total functions from T_1 to T_2:

$$T_1 \rightarrow T_2 == \{f : T_1 \nrightarrow T_2 \mid \operatorname{dom} f = T_1\}$$

In a similar way, define set comprehensions to give:

(a) the set of partial injections from T_1 to T_2;

(b) the set of total injections from T_1 to T_2;

(c) the set of partial surjections from T_1 to T_2;

(d) the set of total surjections from T_1 to T_2;

(e) the set of bijections from T_1 to T_2.