

Bayesian Deep Learning

Yarin Gal

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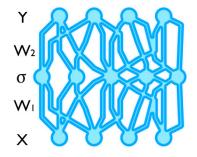


Conceptually simple models

Data: $\mathbf{X} = {\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_N}, \mathbf{Y} = {\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_N}$ **Model**: given matrices **W** and non-linear func. $\sigma(\cdot)$, define "network"

$$\tilde{\mathbf{y}}_i(\mathbf{x}_i) = \mathbf{W}_2 \cdot \sigma(\mathbf{W}_1 \mathbf{x}_i)$$

Objective: find **W** for which $\tilde{\mathbf{y}}_i(\mathbf{x}_i)$ is close to \mathbf{y}_i for all $i \leq N$.





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Deep learning is awesome

- Simple and modular
- Huge attention from practitioners and engineers
- Great software tools
- Scales with data and compute
- Real-world impact

- ... but has many issues 🗙
 - What does a model not know?
 - Uninterpretable black-boxes
 - Easily fooled (AI safety)
 - Lacks solid mathematical foundations (mostly ad hoc)
 - Crucially relies on big data



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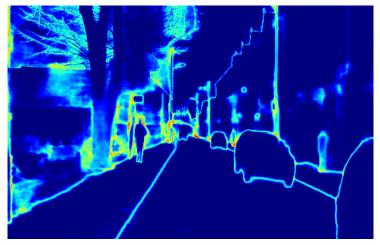


- We need a way to tell what our model knows and what not.
 - We train a model to recognise dog breeds
 - And are given a cat to classify
 - What would you want your model to do?
 - ► Similar problems in *decision making*, *physics*, *life science*, etc.





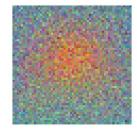
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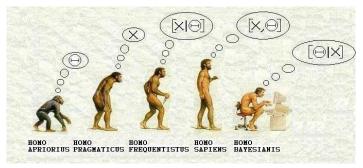


The language of uncertainty

- Probability theory
- ► Specifically Bayesian probability theory (1750!)

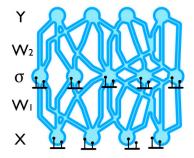
When applied to Information Engineering...

Bayesian modelling

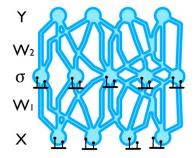


- Built on solid mathematical foundations
- Orthogonal to deep learning...

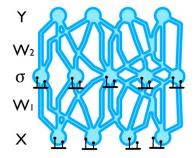
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- Works by randomly setting network units to zero
- ► This somehow improves performance and reduces over-fitting
- ▶ Used in almost all modern deep learning models



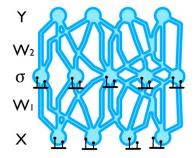
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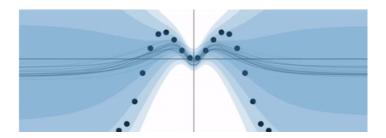
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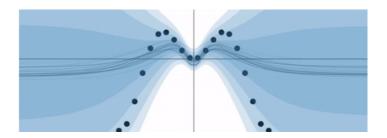


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- Connecting **Deep Learning to Bayesian probability theory**.
- The mathematically grounded connection gives a treasure trove of new research opportunities:
 - uncertainty in deep learning, e.g. interpretability and Al safety
 - principled extensions to deep learning
 - enable deep learning in small data domains

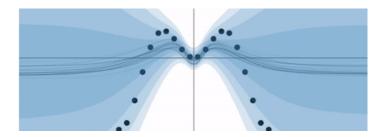


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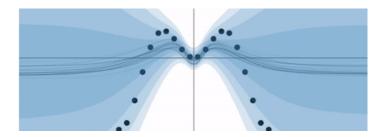
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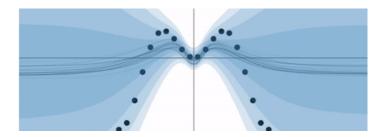
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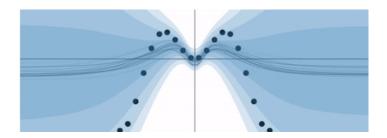
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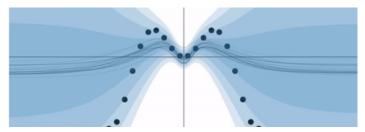


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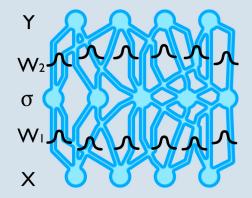
More in a second. First, some theory.





From Bayesian neural networks to Dropout

▶ Place **prior** p(W) dist. on weights, making these r.v.s



• Given dataset **X**, **Y**, the r.v. **W** has a **posterior**: $p(\mathbf{W}|\mathbf{X}, \mathbf{Y})$



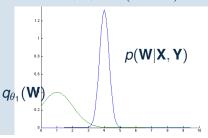
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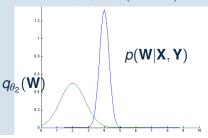


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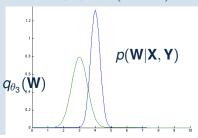


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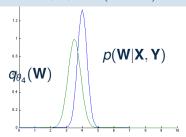


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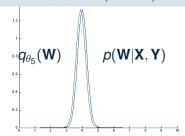


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Theorem (Dropout as approximate variational inference)

Define

 $q_{\mathbf{M}}(\mathbf{W}) := \mathbf{M} \cdot diag(Bernoulli)$

with variational parameter M.

The optimisation objective of (stochastic) variational inference with $q_{\mathbf{M}}(\mathbf{W})$ is identical to the objective of a dropout neural network.

Proof.

See Gal [2016].



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Implementing **inference** with $q_{\mathbf{M}}(\mathbf{W})$

Implementing **dropout training**. Line to line.



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Corollary (Model uncertainty with dropout)

Given $p(\mathbf{y}^*|\mathbf{f}^{\mathbf{W}}(\mathbf{x}^*)) = \mathcal{N}(\mathbf{y}^*; \mathbf{f}^{\mathbf{W}}(\mathbf{x}^*), \tau^{-1}\mathbf{I})$ for some $\tau > 0$, the model's predictive variance can be estimated with the unbiased estimator:

$$\widetilde{Var}[\mathbf{y}^*] := \tau^{-1}\mathbf{I} + \frac{1}{T}\sum_{t=1}^T \mathbf{f}^{\widehat{\mathbf{W}}_t}(\mathbf{x}^*)^T \mathbf{f}^{\widehat{\mathbf{W}}_t}(\mathbf{x}^*) - \widetilde{\mathbb{E}}[\mathbf{y}^*]^T \widetilde{\mathbb{E}}[\mathbf{y}^*]$$

with $\widehat{\mathbf{W}}_t \sim q^*_{\mathbf{M}}(\mathbf{W})$.

Some code, just for fun



In practical terms¹, given point *x*:

- drop units at test time
- repeat 10 times
- ▶ and look at mean and sample variance.
- Or in Python:

```
1 y = []
2 for _ in xrange(10):
3 y.append(model.output(x, dropout=True))
4 y_mean = numpy.mean(y)
5 y_var = numpy.var(y)
```

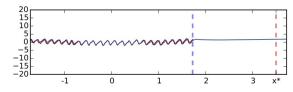
¹Friendly introduction given in yarin.co/blog

Example uncertainty in deep learning



What would be the CO_2 concentration level in Mauna Loa, Hawaii, in 20 years' time?

Normal dropout:



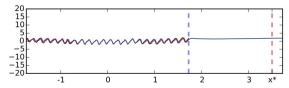
Same network, Bayesian perspective:

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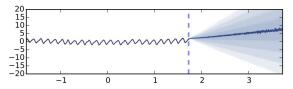


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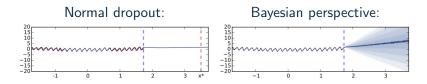


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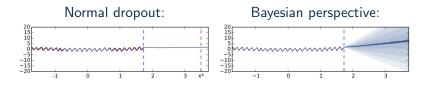


What can we do with this?

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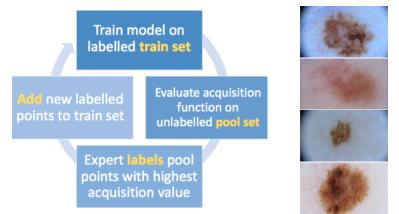


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- Interpretability & AI safety
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 - Cancer diagnosis

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Active learning of images [Gal, Islam & Ghahramani, 2017] E.g. diagnose melanoma with a handful of images.



Active Learning acquisition functions

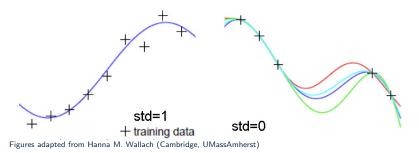


Choose x^* that maximises acquisition functions $a(\mathbf{x})$:

$$\mathbf{X}^* = \operatorname{argmax}_{\mathbf{X} \in \mathcal{D}_{\mathsf{pool}}} a(\mathbf{X})$$

E.g. points that maximise uncertainty. But, which uncertainty?

- ► Aleatoric uncertainty captures noise inherent in the data
- Epistemic uncertainty captures model's lack of knowledge
- Predictive uncertainty captures the sum of the two





Choose x^* that maximises acquisition functions $a(\mathbf{x})$:

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Possible measures of uncertainty in classification:

• Predictive entropy $(\mathbb{H}[y|\mathbf{X}, \mathcal{D}_{train}])$

$$a_{\mathsf{PE}}(\mathbf{x}) = -\sum_{c} p(y = c | \mathbf{x}, \mathcal{D}_{\mathsf{train}}) \log p(y = c | \mathbf{x}, \mathcal{D}_{\mathsf{train}})$$

► Information gained about the model parameters $(I[y, W|x, D_{train}])$

$$a_{\mathsf{MI}}(\mathbf{x}) = \mathbb{H}[y|\mathbf{x}, \mathcal{D}_{\mathsf{train}}] - \mathbb{E}_{p(\mathbf{W}|\mathcal{D}_{\mathsf{train}})} \big[\mathbb{H}[y|\mathbf{x}, \mathbf{W}]\big]$$

Variation ratios

$$a_{\mathrm{VR}}(\mathbf{x}) = 1 - \max_{y} p(y|\mathbf{x}, \mathcal{D}_{\mathrm{train}})$$

• Random acquisition (baseline): $a_U(\mathbf{x}) = unif()$

Acquisition functions intuition

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Want to classify dogs vs. cats given image \bm{x} with models $\mathcal{M}_1, \mathcal{M}_2, \mathcal{M}_3$

- Stochastic forward passes give probability vectors for each model:
 - $1. \ (1,0),...,(1,0)$
 - 2. (0.5, 0.5), ..., (0.5, 0.5), and
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What's the epistemic uncertainty for each model? What's the predictive uncertainty for each model?



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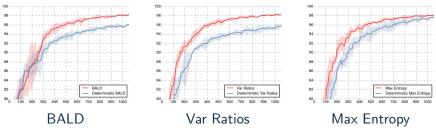
Acquisition functions intuition:

- \blacktriangleright $\mathcal{M}_1:$ all acquisition functions give low uncertainty
- ► M₂: variation ratios and predictive entropy give high uncertainty; mutual information gives low uncertainty.
- $\blacktriangleright~\mathcal{M}_3:$ all acquisition functions give high uncertainty

MNIST experiments



Test accuracy as a function of number of acquired images (up to 1K):



using both a Bayesian CNN (red) and a deterministic CNN (blue)

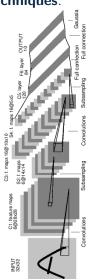
Number of acquired images to get to model error of %:

% error	BALD	Var Ratios	Max Ent	Random
10%	145	120	165	255
5%	335	295	355	835

Active learning vs. semi-supervised learning **P**UNIVERSITY O

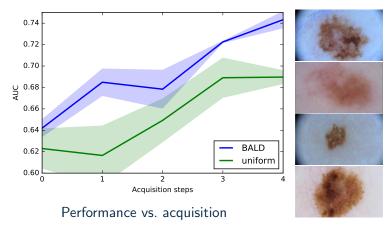
Test error on MNIST with 1000 labelled training samples, for active learning (using simple LeNet) vs. **semi-supervised techniques**:

Technique	Test error			
Semi-supervised:				
SS Embedding (Weston et al., 2012)	5.73%			
DGN (Kingma et al., 2014)	2.40%			
Γ Ladder Network (Rasmus et al., 201	5) 1.53%			
Virtual Adversarial (Miyato et al., 201	5) 1.32%			
Active learning with various acquisitions:				
Random	4.66%			
BALD	1.80%			
Max Entropy	1.74%			
Var Ratios	1.64%			



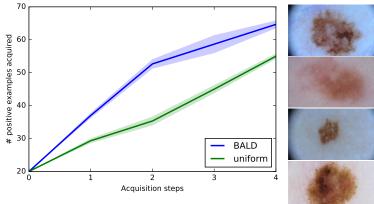
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acquired positive examples vs. acquisition

New horizons





Most exciting is work to come:

- ▶ What is *interesting* data to **label**? (when model is uncertain)
- ► Active learning in real-world medical applications

and much, much, more.

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Thank you for listening.

References



- ▶ Y Gal, R Turner, "Improving the Gaussian process sparse spectrum approximation by representing uncertainty in frequency inputs", ICML (2015)
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