Mathematical Foundations of Bidirectional Transformations

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Take home opportunities

Exercises

You don't have to do them, but if you want to learn this stuff you need to play

Would you like some?

Exercises

1. "Prove" the folk theorem that g is surjective in any asymmetric set-based lens satisfying the three axioms. Which axioms do you really need in your proof?
2. Now fix it — the folk theorem is false. Why? State the correct theorem, and check that your proof is really a proof now.
3. What is the free category on a directed graph? Compare it with the free monoid on a set. How is the free category a "set of terms"?
4. What is the free category on a directed graph which has only one object? It's something you know already.

Exercises continued

5. First, describe a finite monoid M of your choice. The counit of the free monoid adjunction $F \dashv U$ is a monoid morphism from the free monoid on the elements of M to M itself. Describe the effect of the counit in your case explicitly. If M did not have cardinality greater than 1 then do the question again.
6. Show that, as claimed, $\Sigma \dashv \Delta$. Is that true even when V is empty?
7. (After Friday's lecture): State the axioms PutGet, GetPut and PutPut for d-lenses.
8. Repeat the previous question for c-lenses.

Exercises continued further

9. Prove that if

$$\begin{tikzcd} A \arrow[r, "j"] \arrow[d, "j"'] \arrow[dr, phantom, "\text{PULLBACK}"] & B \arrow[d, "g"] \\ B \arrow[r, "g"'] & C \end{tikzcd}$$

   is a pullback, then g is monic (that is, whenever $gh = gk$ it necessarily follows that $h = k$).
   This is how pullback squares can be used to specify that arrows in EA-sketches are required to be monic in models.

10. Prove that a natural transformation between models of keyed EA-sketches has components which are all monic.
    This demonstrates that keyed databases have categories of models which are partial orders (there exists at most one arrow (one update) between any two states). The ordering is the *information order*.

Exercises continued further further

11. Choose your favourite kind of asymmetric lens. Consider the pullback of the Gets of two such lenses and show how the given lenses determine lens structures on the pullback projections.
(Hint: Use the fact that a pullback in **set** or **cat** can be represented as a collection of pairs (of elements, or of objects and of arrows, respectively)).