# Logical pre- and post-selection paradoxes are proofs of contextuality

Matthew F. Pusey

joint work with Matthew S. Leifer

# The three box paradox is a proof of contextuality

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# Three boxes

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Post-select $|1\rangle + |2\rangle - |3\rangle$

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or "Look in box 2":
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# Kochen-Specker non-contextuality

1. *Outcome determinism for projective measurements*: One outcome of a projective measurement is assigned probability $1$, the rest $0$.
2. *Measurement non-contextuality for projective measurements*: The assignment to a projector is independent of the other outcomes in the measurement.

# Avoiding KS contextuality


quant-ph/0412179

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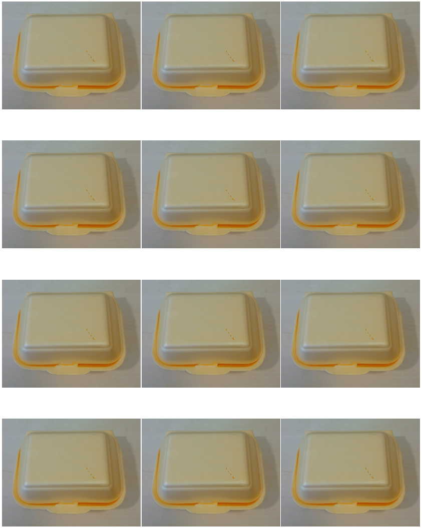

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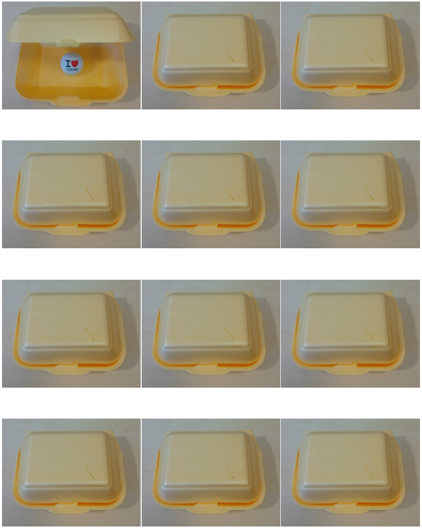

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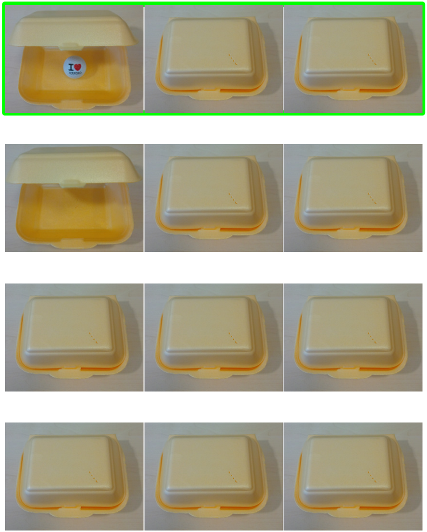

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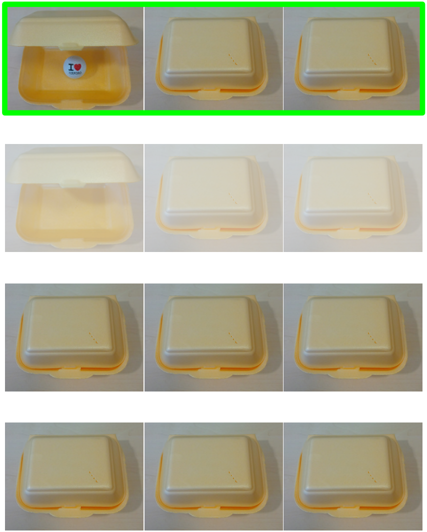

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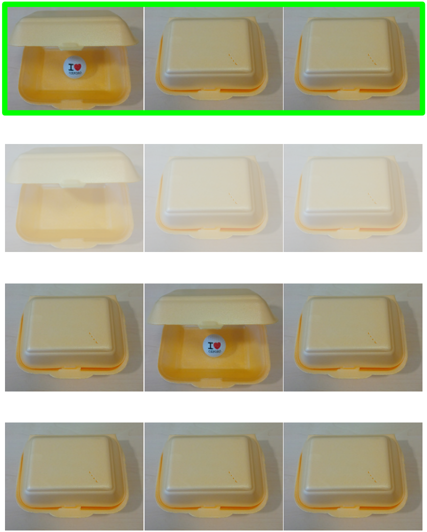

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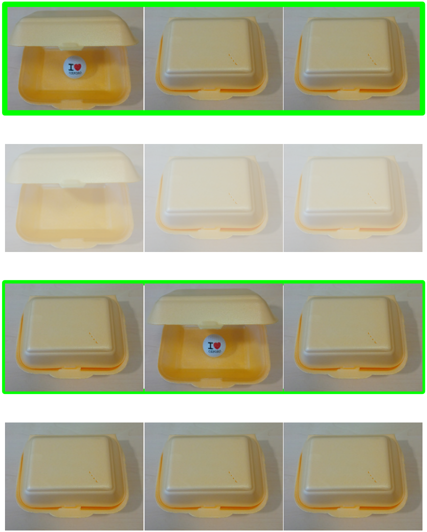

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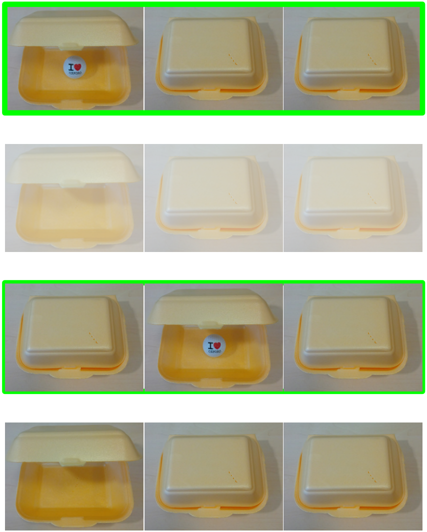

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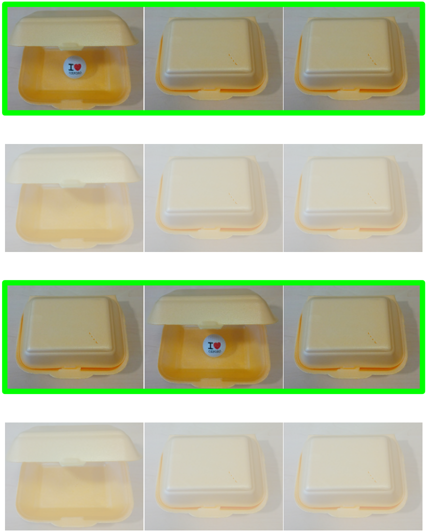

# Avoiding KS contextuality

arXiv:1207.3114

# Relation to KS contextuality

quant-ph/0412178

# Generalised non-contextuality[1]

If two procedures are equivalent at the *operational* level, then they are equivalent at the *ontological* level.

[1] quant-ph/0406166

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If two procedures are equivalent at the *operational* level, then they are equivalent at the *ontological* level.
"Procedures" encompasses preparations, transformations and measurements.

[1] quant-ph/0406166

# Necessity of disturbance

# Two equivalent transformations

$$P\rho P + Q\rho Q$$

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$$= \frac{1}{2}(P\rho P + Q\rho Q + P\rho P + Q\rho Q)$$

$$= \frac{1}{2}(P\rho P{+}Q\rho P{+}P\rho Q{+}Q\rho Q{-}Q\rho P{-}P\rho Q{+}Q\rho Q)$$

$$= \frac{1}{2}\left((P+Q)\rho(P+Q) + (P-Q)\rho(P-Q)\right)$$

$$= \frac{1}{2}\left(\rho + (P-Q)\rho(P-Q)\right)$$

# Two equivalent transformations

$$
\begin{aligned}
& P\rho P + Q\rho Q \\
&= \frac{1}{2}\left(\rho + U\rho U^{\dagger}\right)
\end{aligned}
$$

where $U = P - Q$.

# Read the paper, arXiv:1506.07850 for. . .

- All logical pre-and post-selection paradoxes (e.g. "quantum pigeonhole principle")
- Measurement non-contextuality instead of transformation non-contextuality
- Weak measurement versions
- Importance of 0/1 probabilities, von-Neumann update rule.