Fermionic quantum theory and superselection rules for operational probabilistic theories

Alessandro Tosini, QUIT group, Pavia University

Joint work with

G.M. D'Ariano, F. Manessi, P. Perinotti

Supported by

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Outline

1. Are fermions systems of the usual quantum theory?
2. Fermions as “bricks” of a new operational probabilistic theory
3. Informational features:
   - tomography in fermionic quantum theory
   - fermionic entanglement
4. A definition of superselection for a general probabilistic theory:
   - fermionic and real QT as special cases
5. Future perspectives

1. Are fermions systems of the usual quantum theory?

Simulating Physics with Computers

Richard P. Feynman

Department of Physics, California Institute of Technology, Pasadena, California 91107

Received May 7, 1981

The question is, if we wrote a Hamiltonian which involved only these operators, locally coupled to corresponding operators on the other space-time points, could we imitate every quantum mechanical system which is discrete and has a finite number of degrees of freedom? I know, almost certainly, that we could do that for any quantum mechanical system which involves Bose particles. I'm not sure whether Fermi particles could be described by such a system. So I leave that open. Well, that's an example of what I meant by a general quantum mechanical simulator. I'm not sure that it's sufficient, because I'm not sure that it takes care of Fermi particles.

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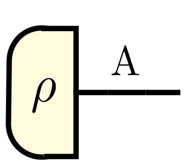
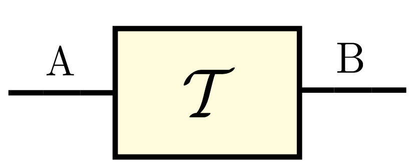
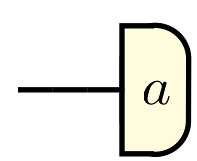


states



maps



measurements

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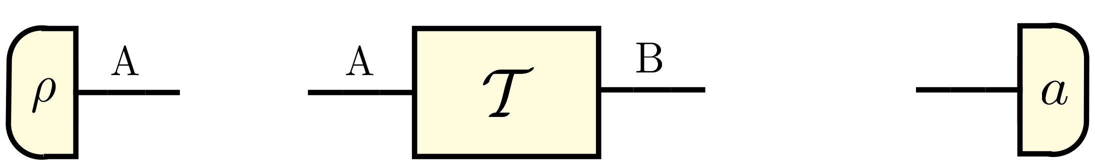
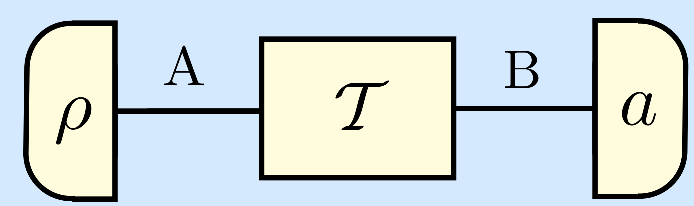


states maps measurements

$$\mathrm{Prob}(\rho, \mathcal{T}, a)$$



Probabilities must
be reproduced correctly

Particle physics

Quantum theory

Particle physics

Quantum theory

Relativity: fermionic field

Fermions are anti-commuting systems

$$\{\varphi_i, \varphi_j\} = 0 \quad \{\varphi_i, \varphi_i^\dagger\} = \delta_{ij} I$$

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$$|\psi\rangle = \cancel{|\text{even particles}\rangle + |\text{odd particles}\rangle}$$

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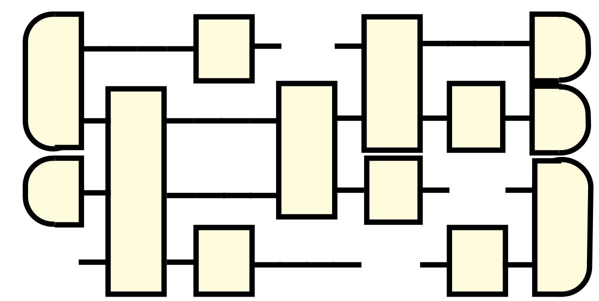

“Qubits” are commuting systems

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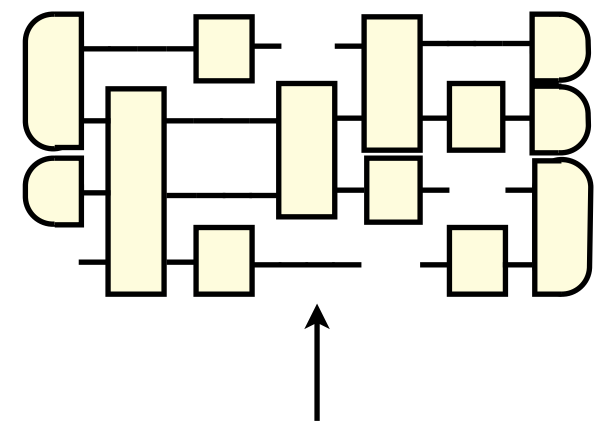

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No a priori *superselection*

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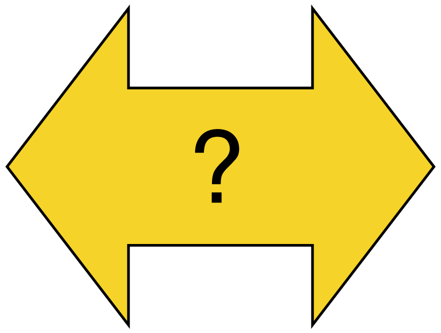

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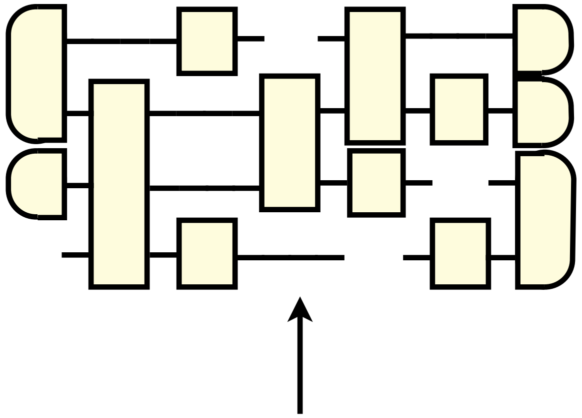

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Usual approach to the problem

Local fermionic mode: system

$$\text{system} \rightarrow \begin{cases} |0\rangle & \text{empty} \\ \varphi^\dagger\,|0\rangle & \text{1 fermion} \end{cases}$$

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N Local Fermionic modes (LFM) $\cong$ *N* qubits

Jordan-Wigner
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Price to pay for anti-commutation

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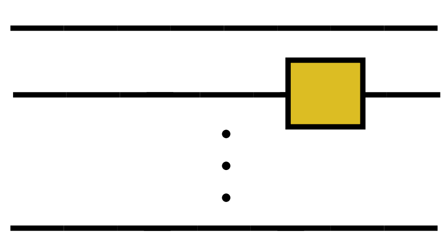
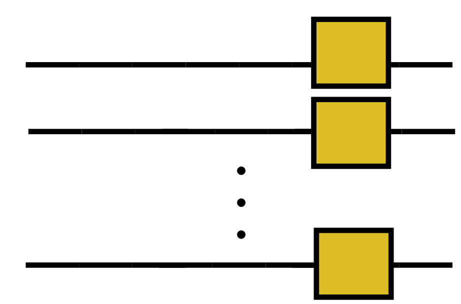


$\xrightarrow{\text{isomorphism}}$

N qubits



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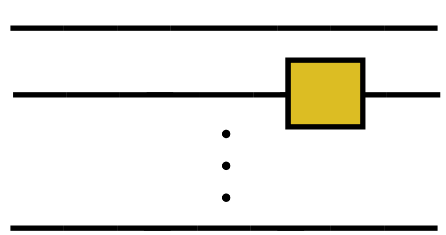
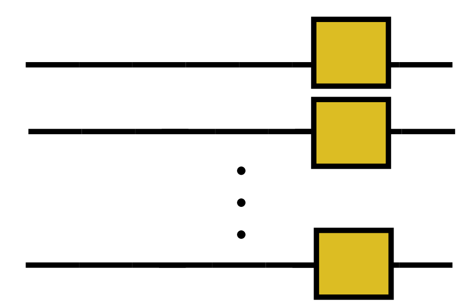


isomorphism $\longrightarrow$

N qubits



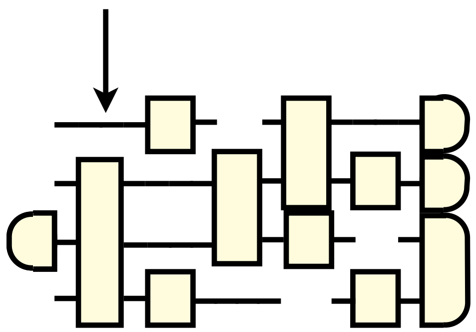
2 Warnings: Where does parity superselection come from?

What do I map via the Jordan-Wigner map?

S. B. Bravyi and A. Y. Kitaev, Annals of Physics 298, 210 (2002)

2. Fermions as “bricks” of a new operational probabilistic theory

Elementary systems:
local fermionic modes



G. M. D’Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)

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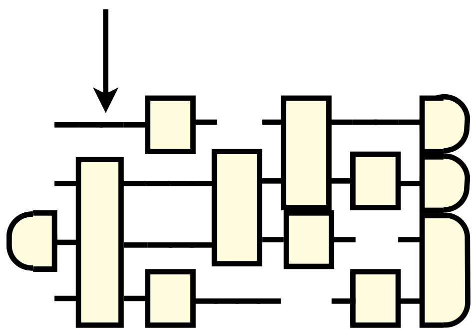

Construction of the theory:

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States and maps in terms of the fields $\varphi_i, \varphi_i^\dagger$

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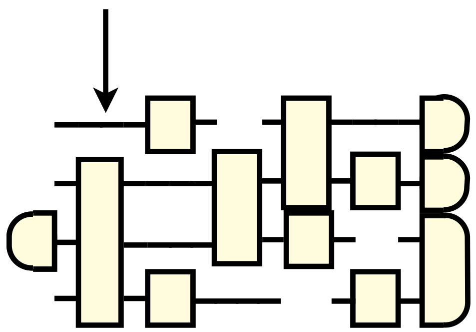

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maps? $\mathcal{T}(\rho) = \sum_i s_i K_i \rho K_i^\dagger$

kraus operator
“field algebra”

states? $\rho := \sum_j K_j |\Omega\rangle \langle\Omega| K_j^\dagger$

vacuum

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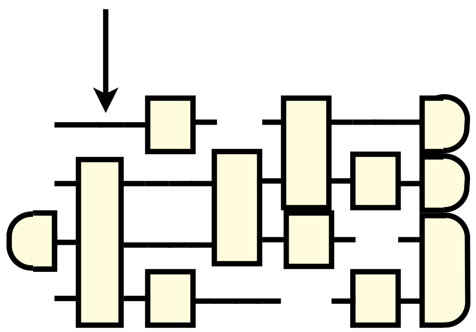

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Operational assumptions:

The field φ must be “physical”: Maps with single Kraus $\alpha\varphi + \beta\varphi^\dagger$ are maps of the theory

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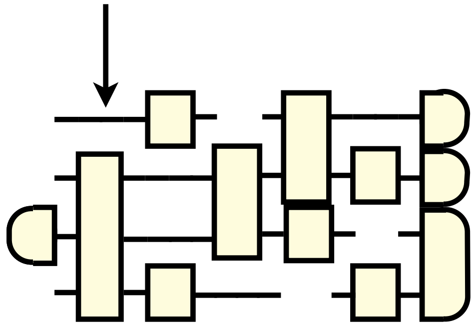

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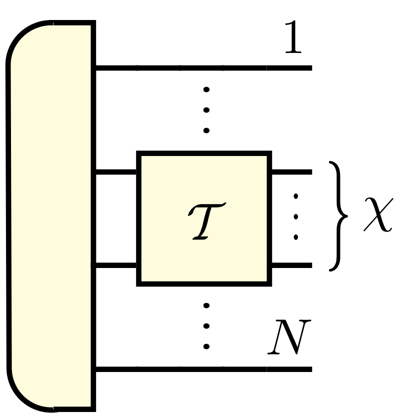
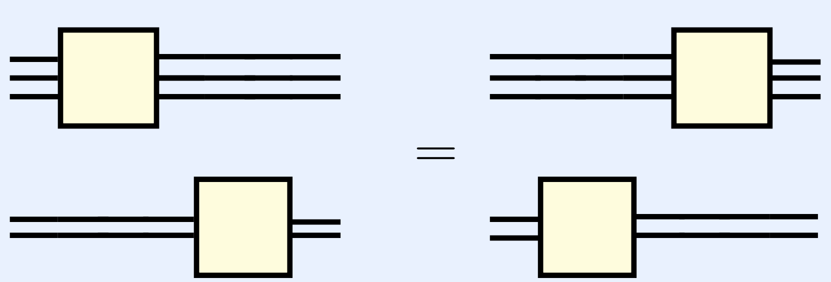
Notion of **local operations**: A map made of fields on some modes => Local on that modes



LOCAL in the operational sense



G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)

Proposition: Fermionic states satisfy the **parity superselection rule**

G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)

Fock space: $\mathcal{F} = \mathcal{F}_e \oplus \mathcal{F}_o$

even sector

odd sector

never superimposed

Any state is of the form:

$$\rho = \left(\begin{array}{c|c} p\rho_e & 0 \\ \hline 0 & (1-p)\rho_o \end{array}\right)$$

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2-LFM:

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$$\cancel{\alpha\,|00\rangle + \beta\,|10\rangle}$$

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NOTICE:
1-LFM = classical bit

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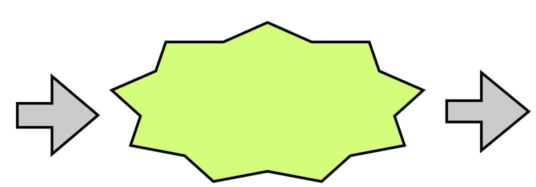
NOTICE:
no conserved quantity

N fermions  *M*≠*N* fermions

Wick, G. C., Wightman, A. S., and Wigner, E. P., 1952, Phys. Rev. 88, 101–105

3. Informational features: tomography for fermionic quantum theory

L. Hardy and W. K. Wootters, Foundations of Physics 42, 454 (2012)

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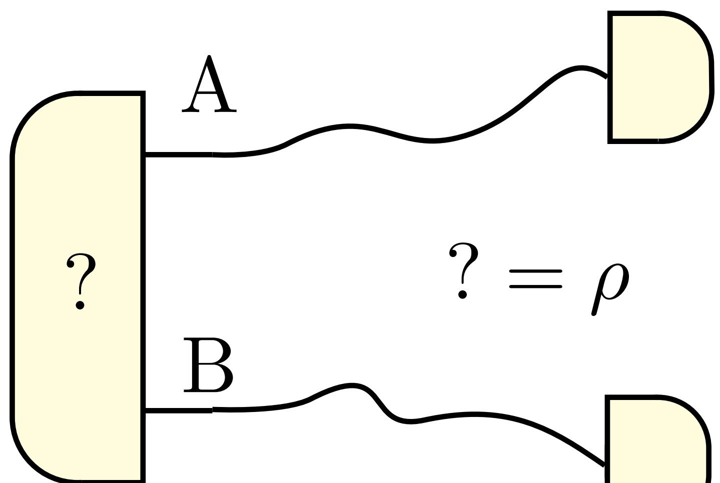
Local tomography:

unknown state



$? = \rho$

Alice

Bob

Alice and Bob determine the state by local measurements

$$D_{\mathrm{AB}} = D_{\mathrm{A}} D_{\mathrm{B}}$$

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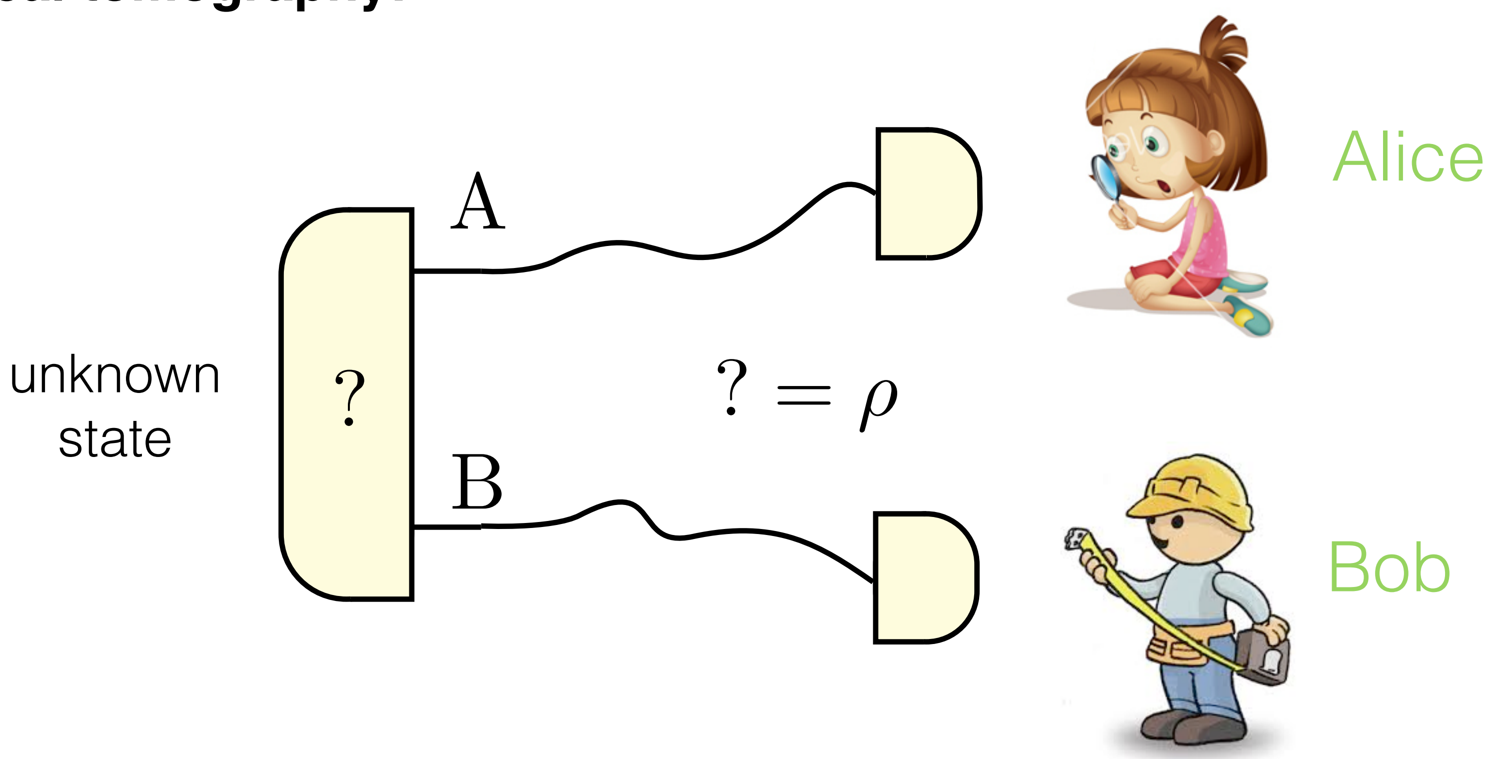

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Non-local tomography: local measurements are not enough

$$D_{\mathrm{AB}} > D_{\mathrm{A}} D_{\mathrm{B}}$$

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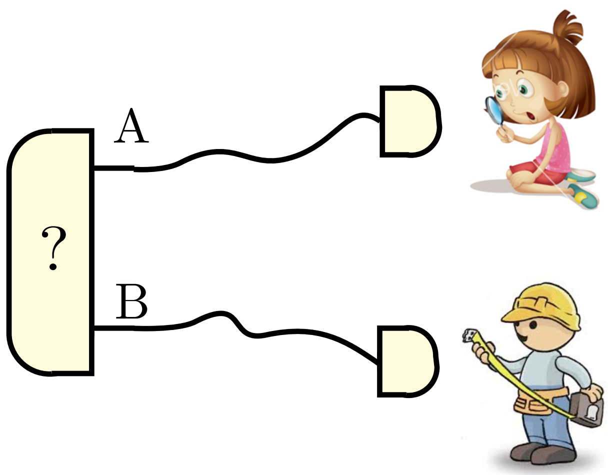
Quantum theory

Fermionic quantum theory



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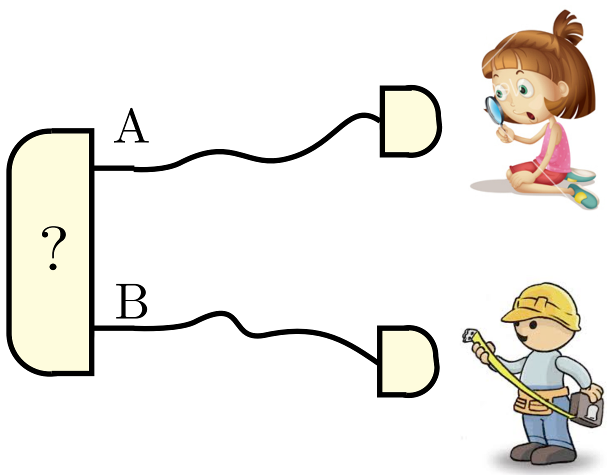

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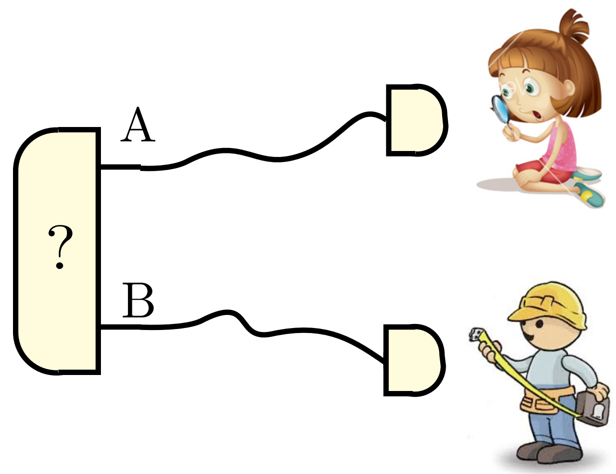

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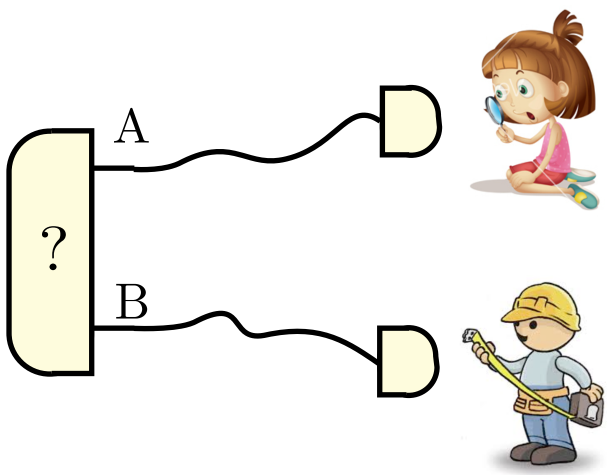

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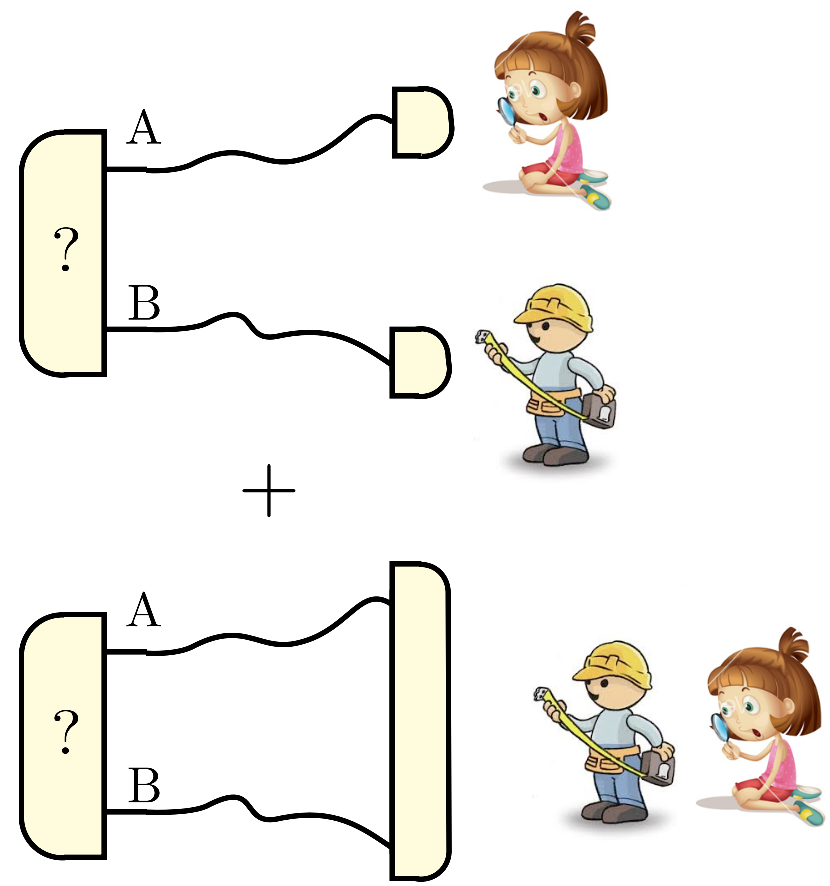

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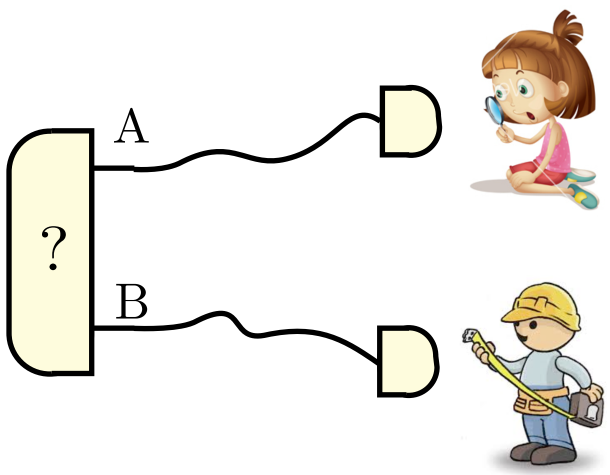

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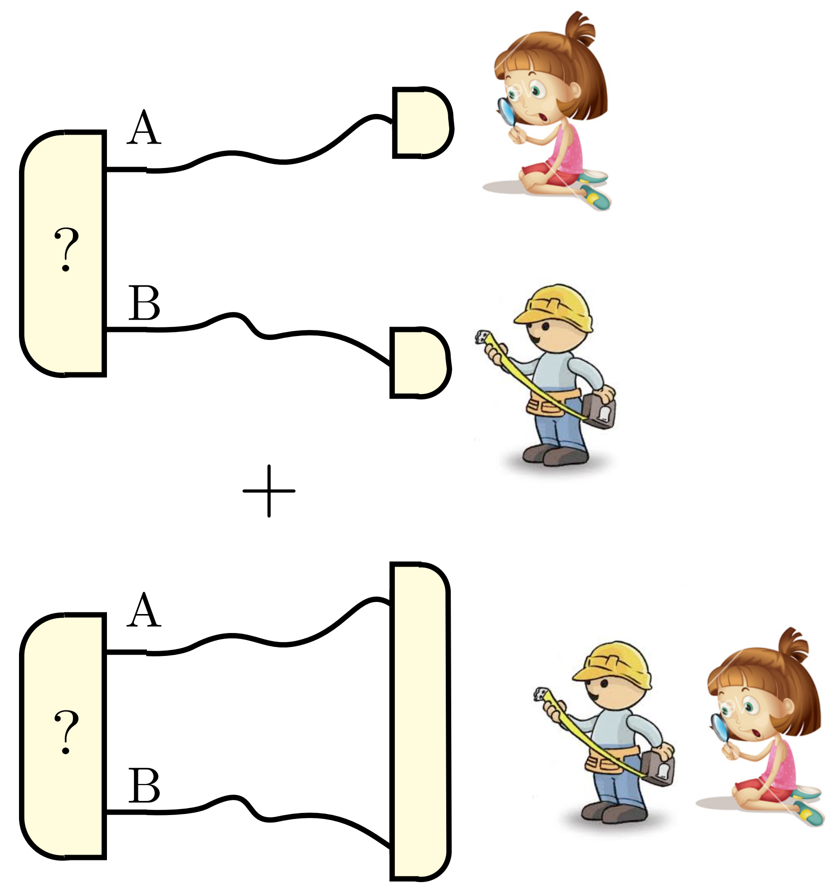

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Bilocal tomography

L. Hardy and W. K. Wootters, Foundations of Physics 42, 454 (2012)

$$D_{\mathrm{AB}} > D_{\mathrm{A}} D_{\mathrm{B}}$$

$$D_{\mathrm{ABC}} \leqslant f(D_{\mathrm{A}}, D_{\mathrm{B}}, D_{\mathrm{C}}, D_{\mathrm{AB}}, D_{\mathrm{AC}}, D_{\mathrm{BC}})$$

NOTICE: this bound is saturated

G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)

3. Informational features: fermionic entanglement

1) Fix a notion of entanglement: **non-separability**
2) Quantify amount of entanglement in operational terms: we choose **Entanglement of formation**

G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)

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Fermionic entanglement of formation

$$|\Psi\rangle_{res}{}^{\otimes N} \xrightarrow{\text{LOCC}} \rho^{\otimes M}$$

N resource states

M copies of ρ

$$E(\rho) = \lim_{M\to\infty} \frac{N(M)}{M}$$

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$E(\rho) = 0 \;\Leftrightarrow\; \rho$ separable

$E(\rho) = 1 \;\Leftrightarrow\; \rho$ maximally entangled

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N resource states

M copies of ρ

$$E(\rho) = \lim_{M\to\infty} \frac{N(M)}{M}$$

$$E(\rho) = 0 \iff \rho \text{ separable}$$
$$E(\rho) = 1 \iff \rho \text{ maximally entangled}$$

Fermionic entanglement of formation

$$|\Psi\rangle_{res}^{\otimes N} \xrightarrow{\text{Ferm. LOCC}} \rho^{\otimes M}$$

$$E_{\mathrm{F}}(\rho) = \lim_{M\to\infty} \frac{N(M)}{M}$$

G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)

3. Informational features: fermionic entanglement

1) Fix a notion of entanglement: **non-separability**
2) Quantify amount of entanglement in operational terms: we choose **Entanglement of formation**

How much entanglement in ρ ?

Quantum entanglement of formation

$$|\Psi\rangle_{res}{}^{\otimes N} \xrightarrow{\text{LOCC}} \rho^{\otimes M}$$

N resource states — M copies of ρ

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$$E_{\mathrm{F}}(\rho) = \lim_{M\to\infty} \frac{N(M)}{M}$$

Proposition:

$$E_{\mathrm{F}}(\rho) \geq p_e E(\rho_e) + p_o E(\rho_o)$$

$$p_e\rho_e + p_o\rho_o$$

G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)

Mixed states with maximal entanglement of formation

$$|\Psi_e\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

$$|\Psi_o\rangle \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)$$

$$\rho_\star = \frac{1}{2}\underbrace{|\Psi_e\rangle\langle\Psi_e|}_{\rho_e} + \frac{1}{2}\underbrace{|\Psi_o\rangle\langle\Psi_o|}_{\rho_o}$$

D'Ariano, G.M., Manessi, F., Perinotti, P., Tosini, A. EPL 107(2), 20,009 (2014)

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As qubits state it has no entanglement

$$E(\rho_\star) = 0$$

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$$

$$\rho_\star = \tfrac{1}{2}|+\rangle\langle+|^{\otimes 2} + \tfrac{1}{2}|-\rangle\langle-|^{\otimes 2}$$

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$$E_{\mathrm{F}}(\overset{p_e\rho_e + p_o\rho_o}{\rho}) \geq p_e E(\rho_e) + p_o E(\rho_o)$$

$$E_{\mathrm{F}}(\rho_\star) \geq \frac{1}{2}\underbrace{E(\rho_e)}_{1} + \frac{1}{2}\underbrace{E(\rho_o)}_{1}$$

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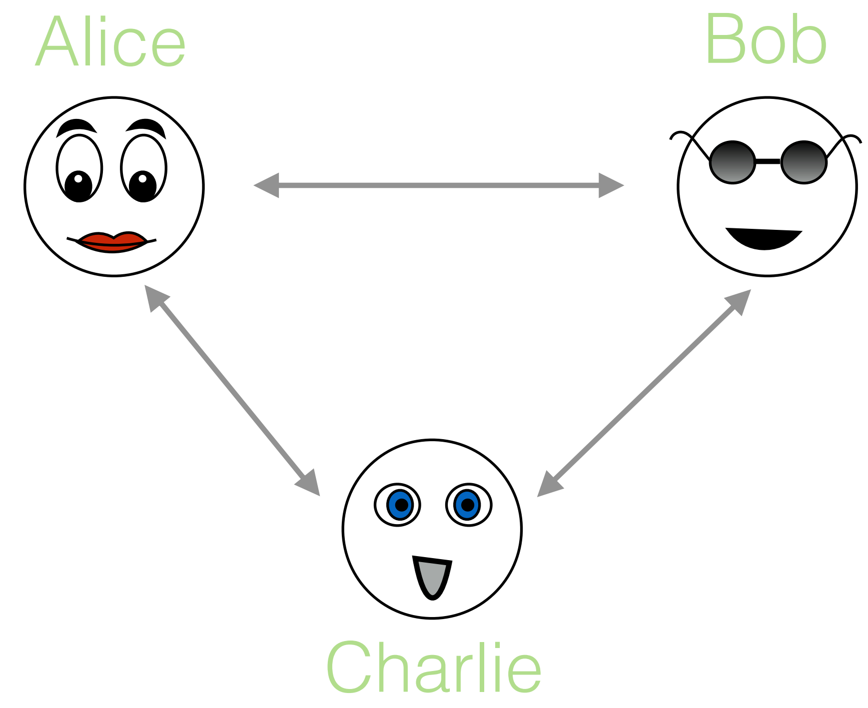
Fermionic entanglement is not monogamous

Quantum entanglement is monogamous

3-qubits: $|\Psi\rangle_{\mathrm{ABC}}$



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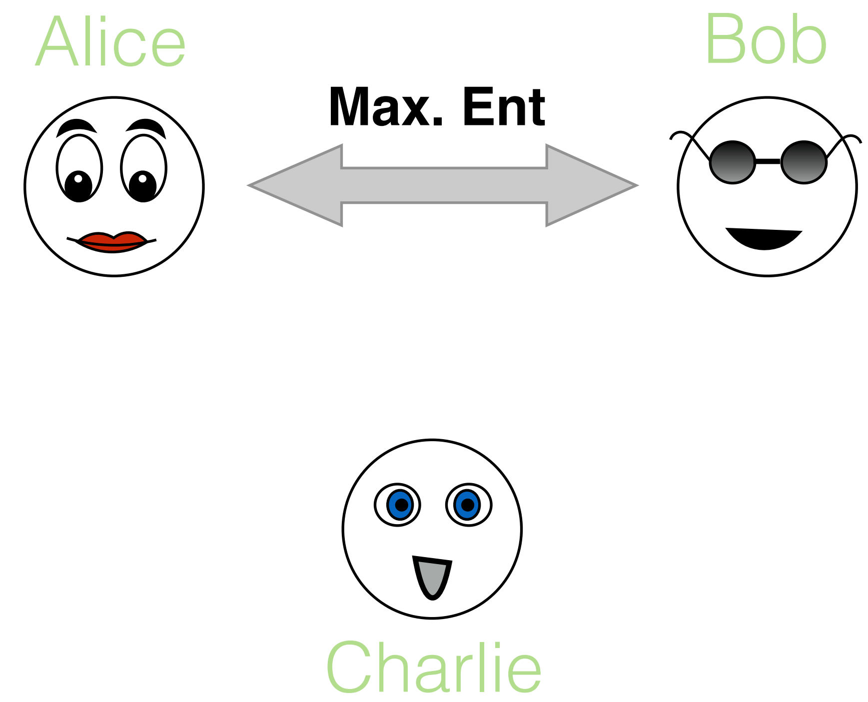

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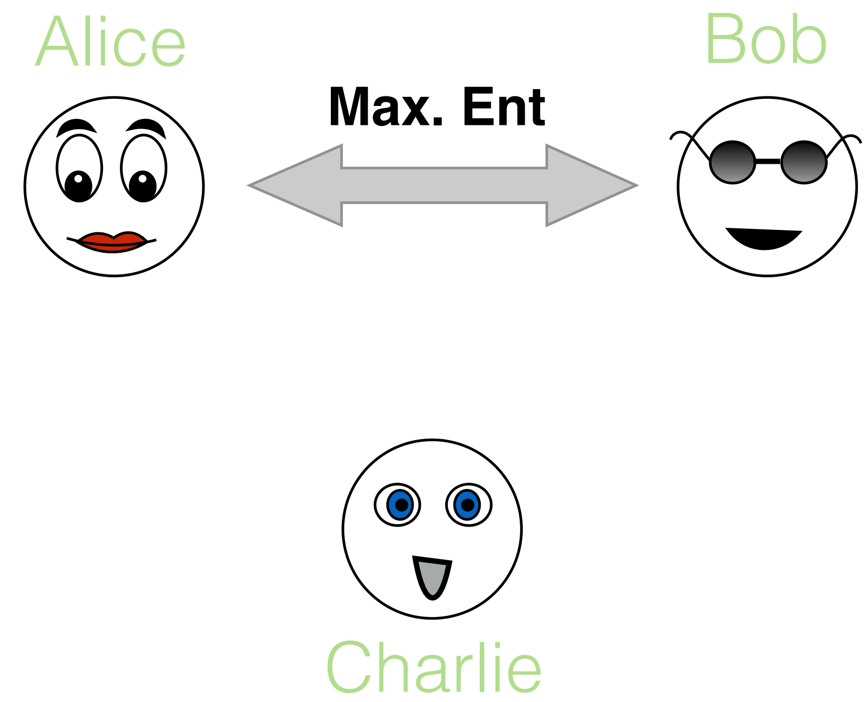
V. Coffman, J. Kundu and W. K. Wootters, Phys. Rev. A 61, 052306 (2000)



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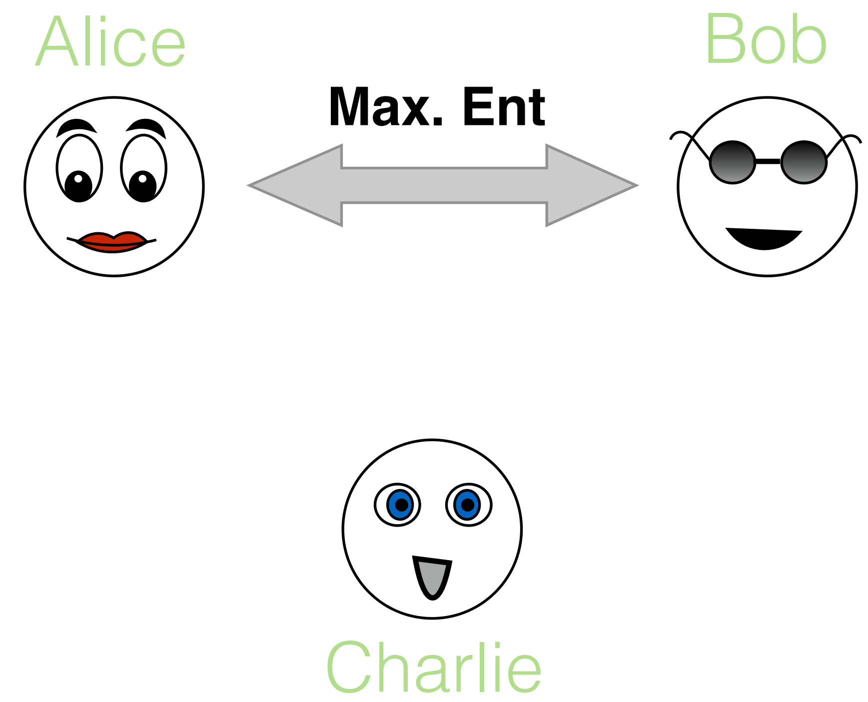

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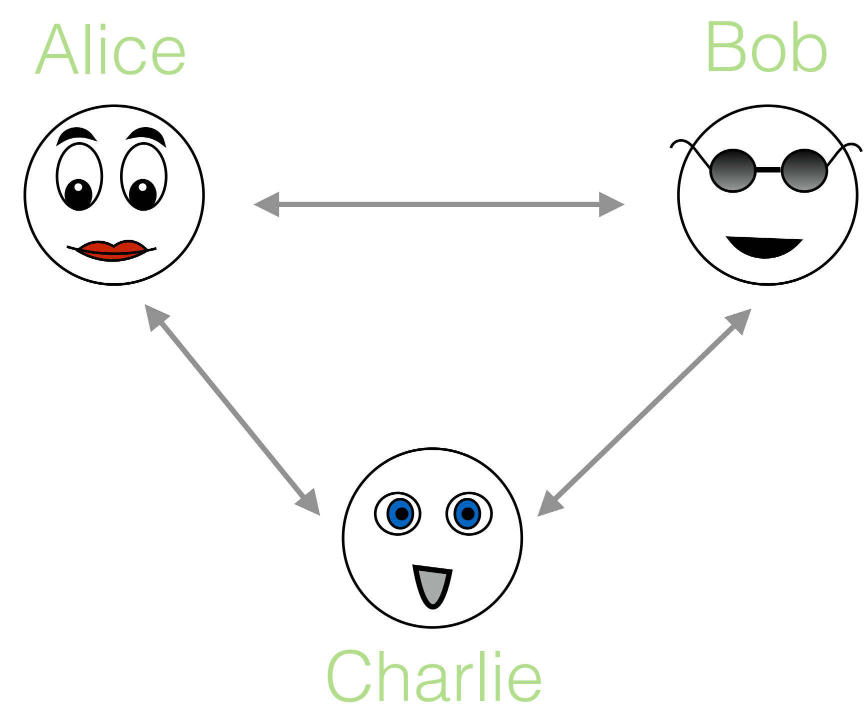

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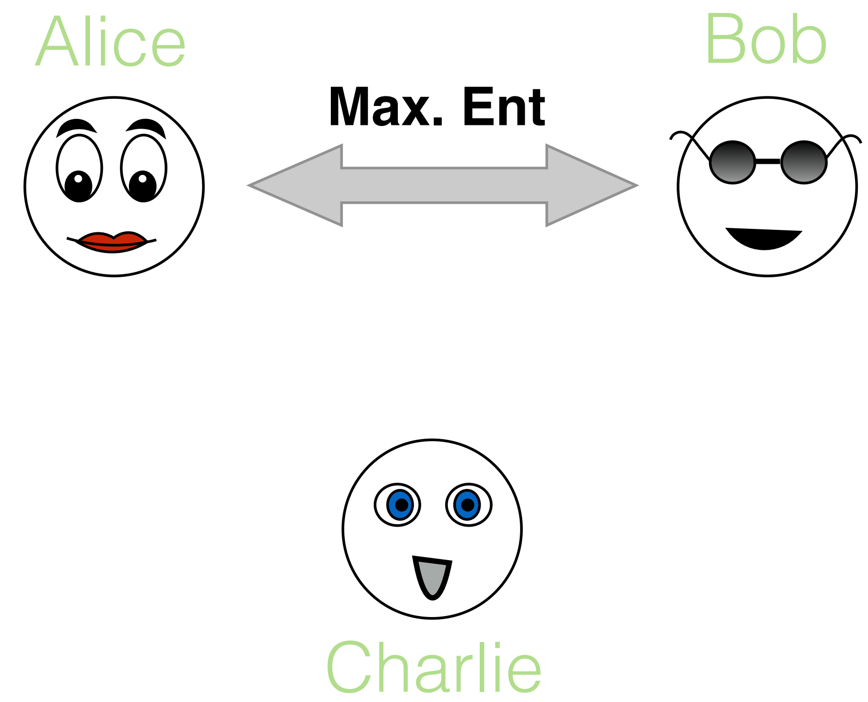

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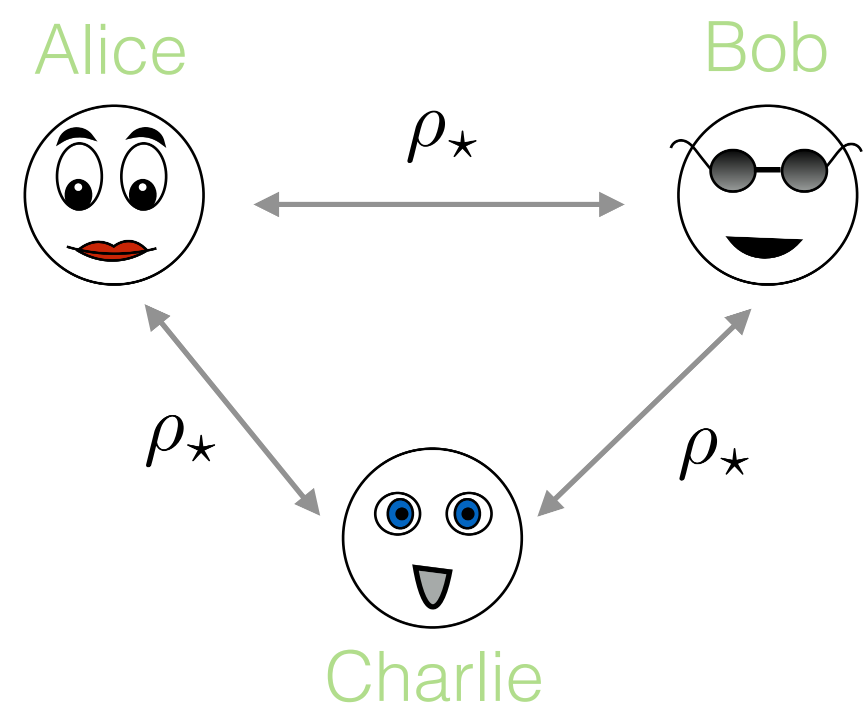

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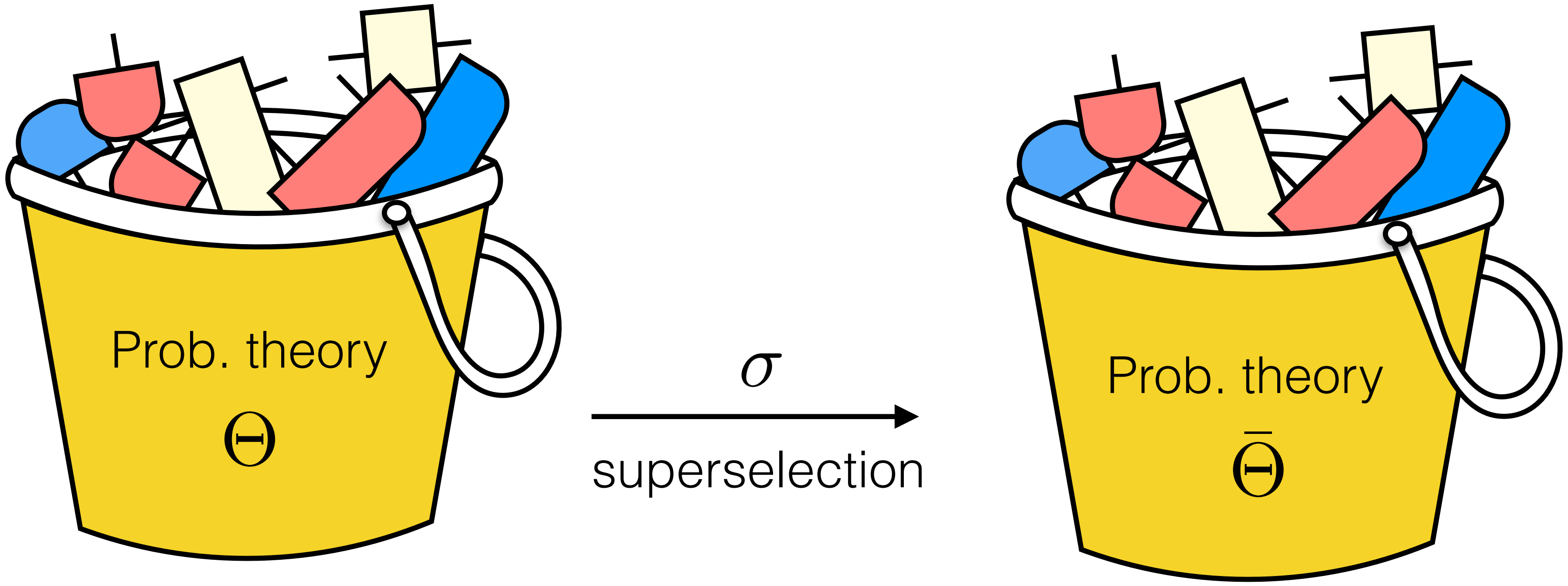
4. Definition of superselection for probabilistic theory



How to define σ **?**

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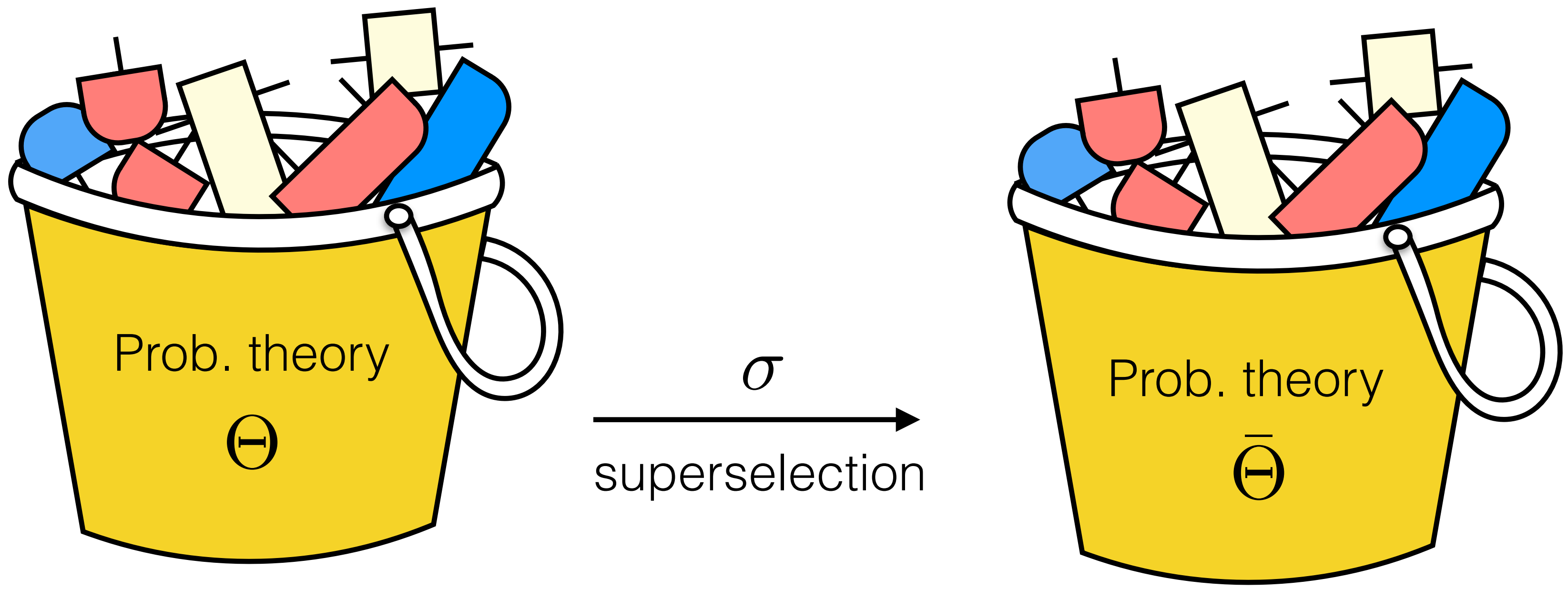

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system $\mathrm{A} \in \Theta$ fully specified by its set of states $\mathrm{St}(\mathrm{A})$

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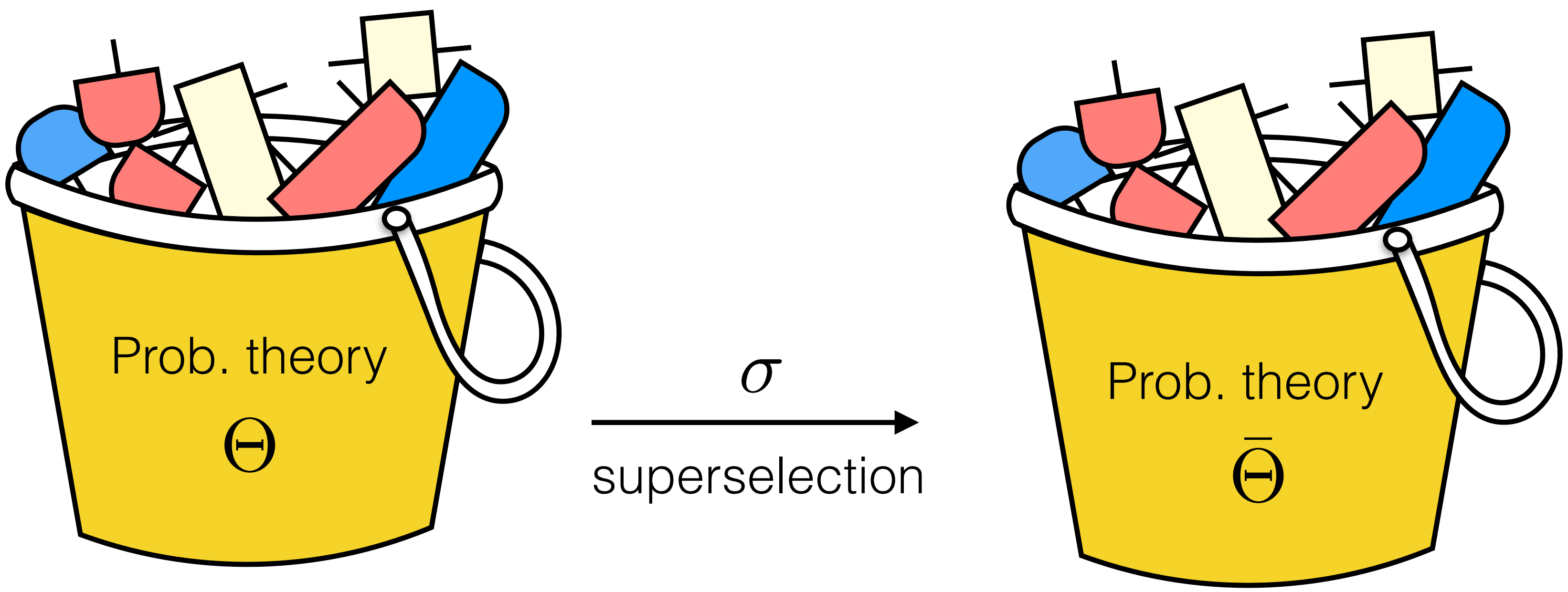

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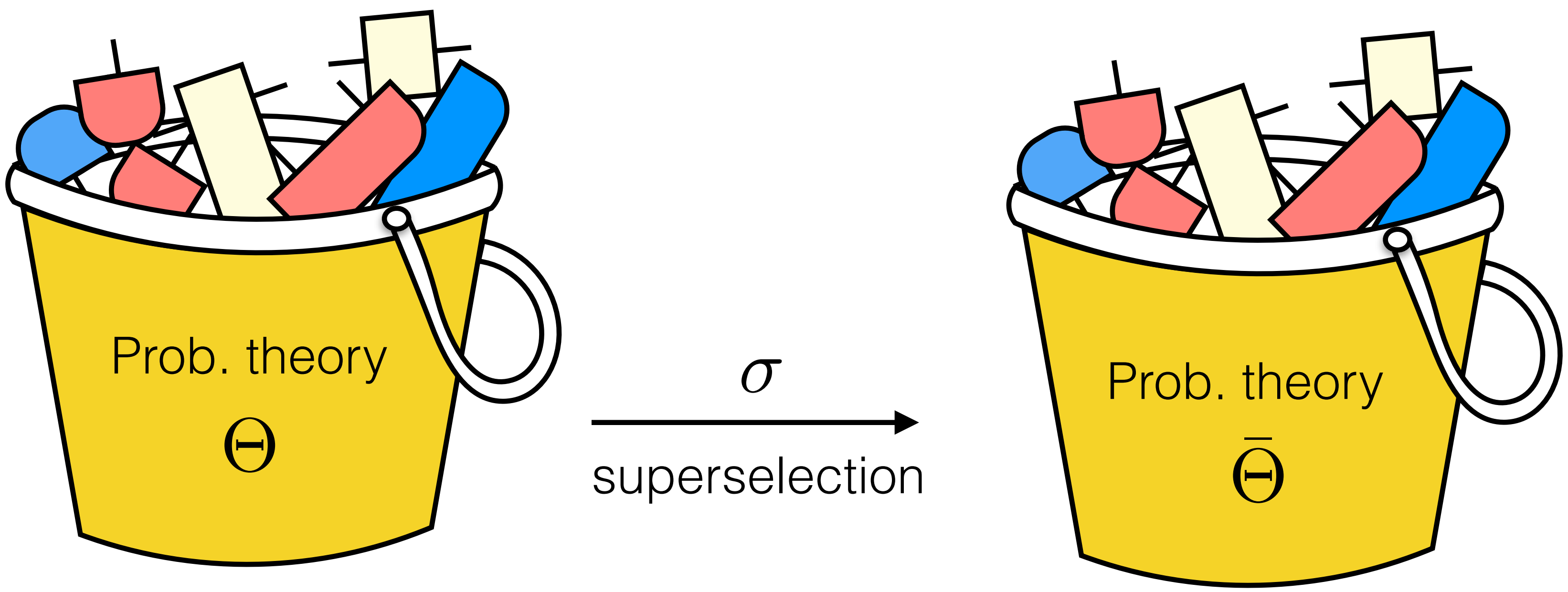

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$$\sigma : \Theta \to \bar{\Theta} \qquad (\mathrm{A}, \mathrm{St}(\mathrm{A})) \mapsto (\bar{\mathrm{A}}, \mathrm{St}(\bar{\mathrm{A}}))$$

$$\mathrm{St}(\bar{\mathrm{A}}) := \{\rho \in \mathrm{St}(\mathrm{A}) \mid (s_i^\sigma|\rho) = 0, \quad i = 1, \ldots, V_{\mathrm{A}}^\sigma\}$$

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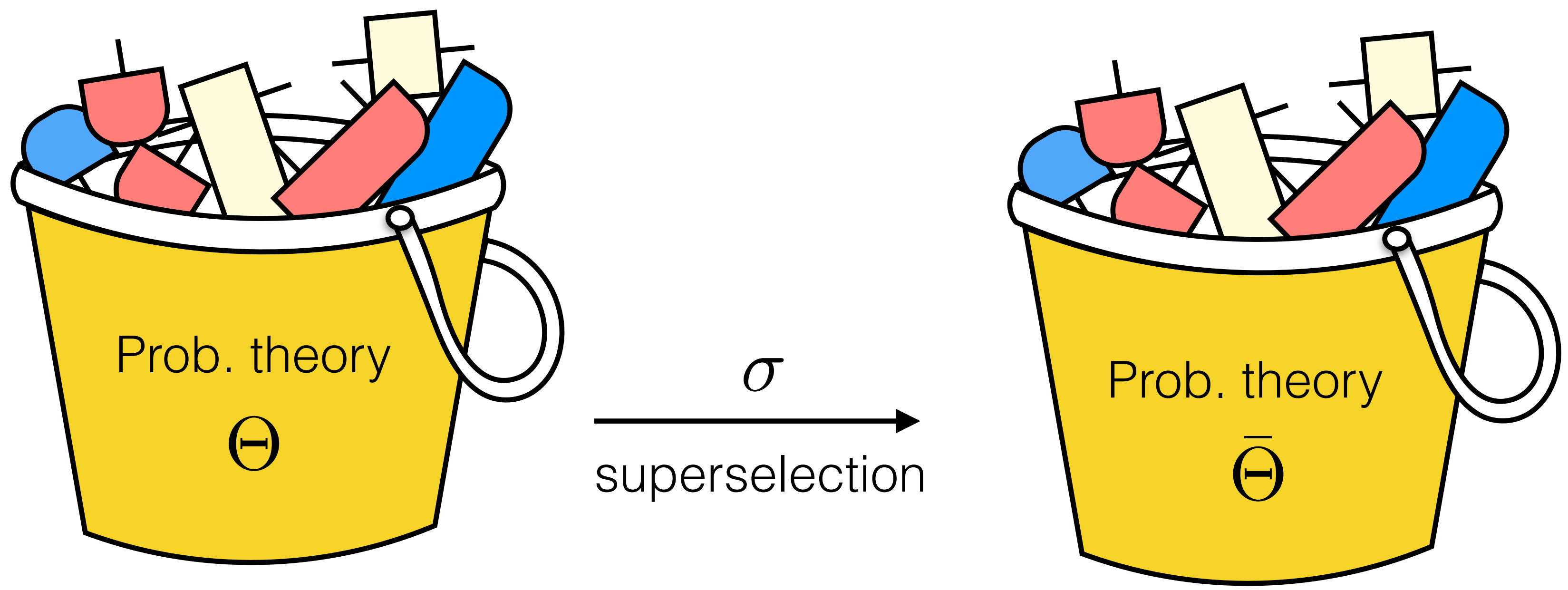

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linear constraint

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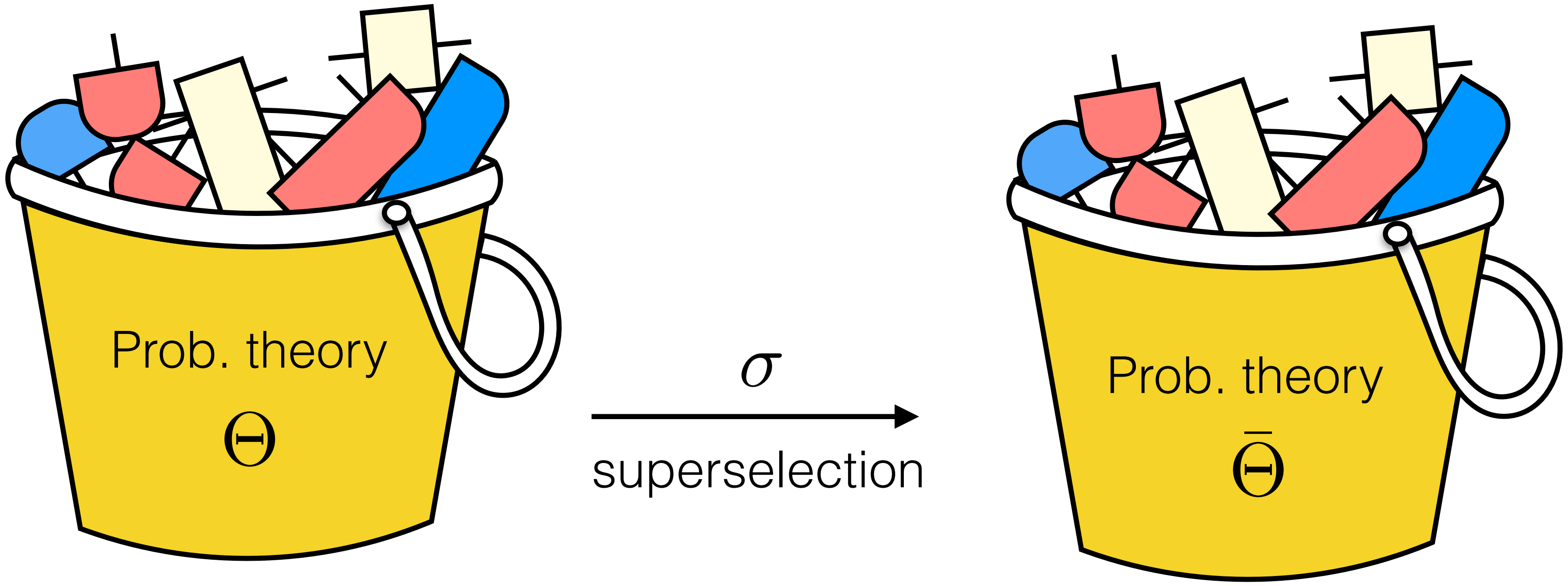

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number of constraints

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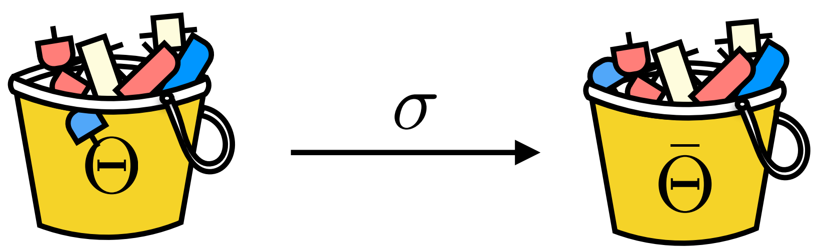
Superselection-holism tradeoff



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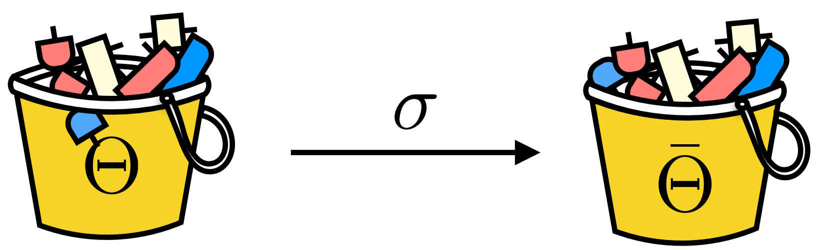

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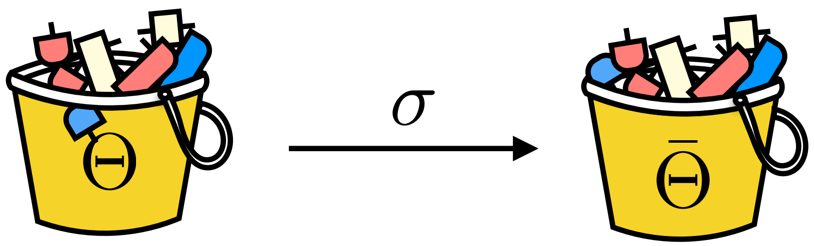
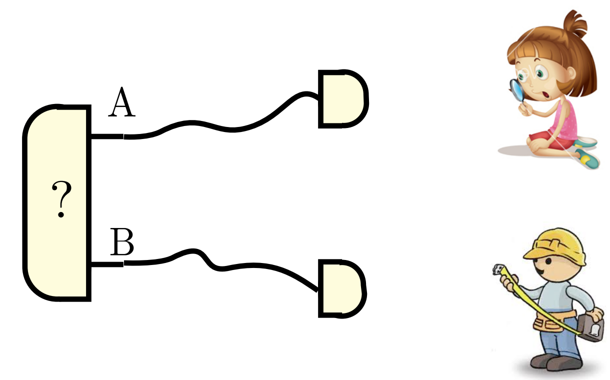


The intuition



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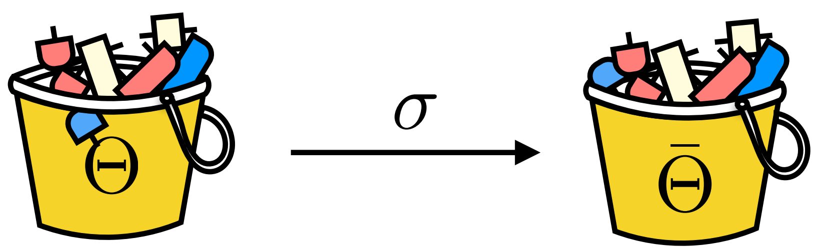

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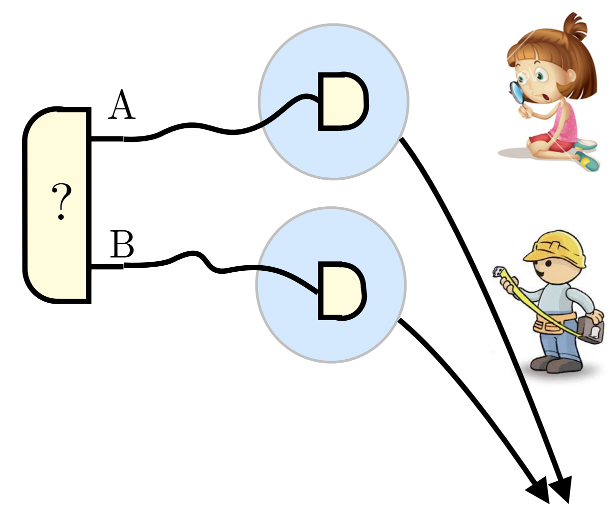

less local measures

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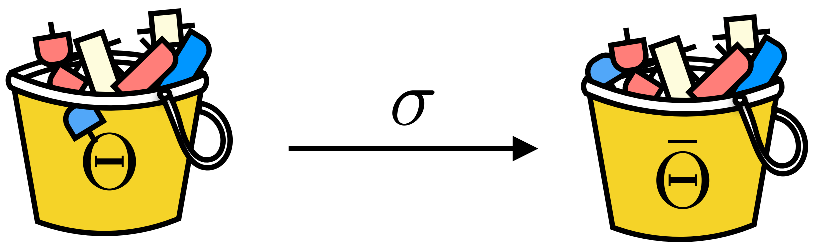

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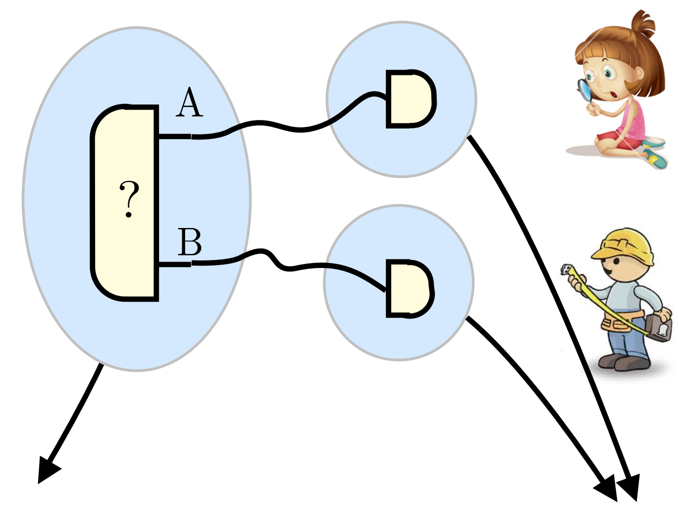

less states

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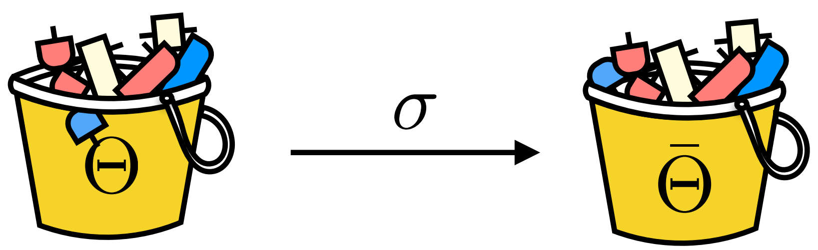

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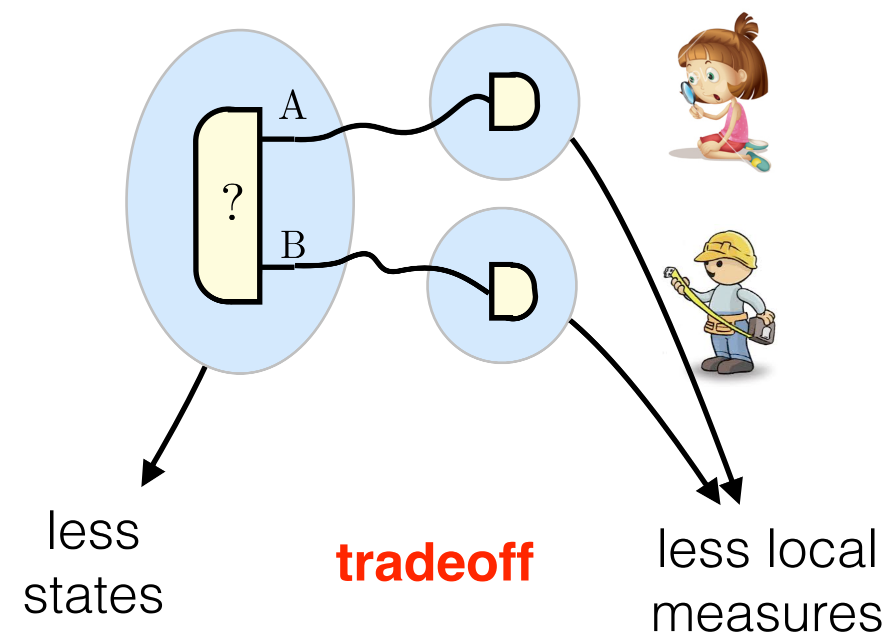

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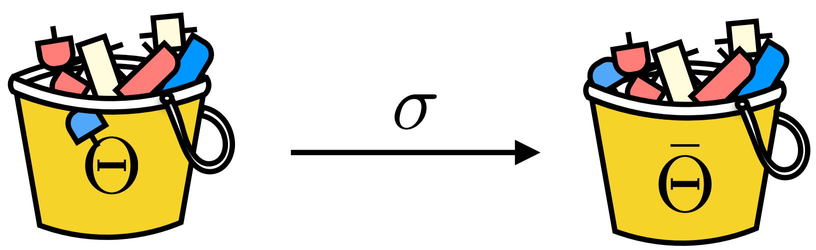

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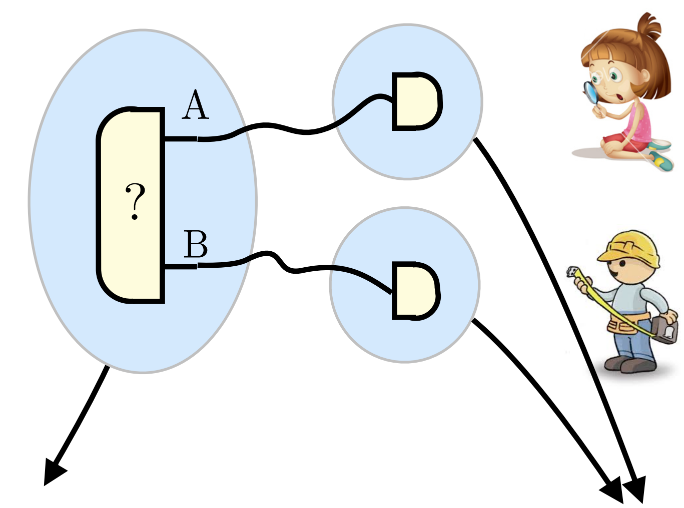

less states

tradeoff

less local measures

Lemma: number of constraints on composite systems

$$L(\sigma, \mathrm{A}, \mathrm{B}) \leqslant V^{\sigma}_{\mathrm{AB}} \leqslant U(\sigma, \mathrm{A}, \mathrm{B})$$

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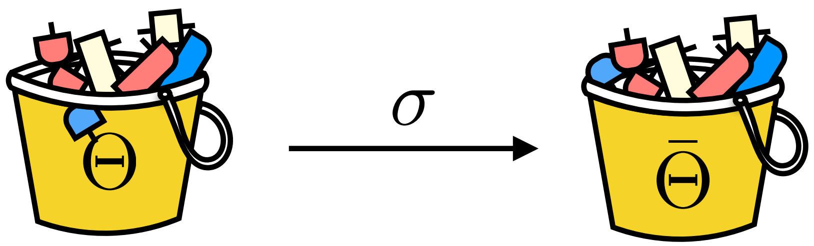

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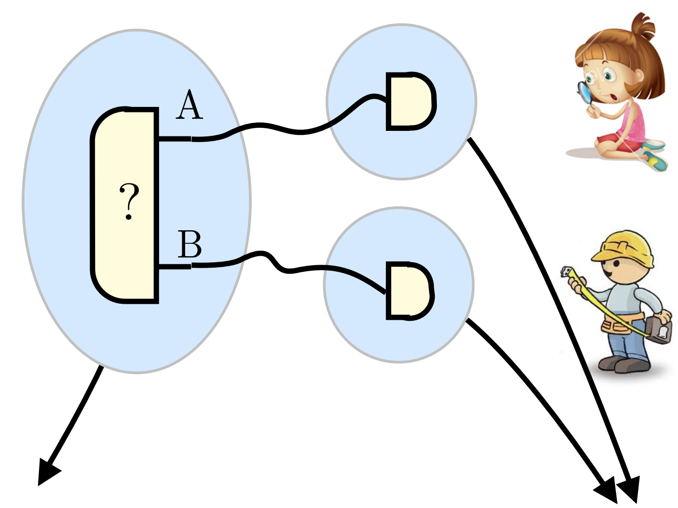

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Lemma: number of constraints on composite systems $\quad L(\sigma, \mathrm{A}, \mathrm{B}) \leqslant V^{\sigma}_{\mathrm{AB}} \leqslant U(\sigma, \mathrm{A}, \mathrm{B})$

Proposition: If Θ satisfies *local tomography* then

i) minimal SSR $\quad V^{\sigma}_{\mathrm{AB}} = L(\sigma, \mathrm{A}, \mathrm{B}) \quad \Rightarrow \quad \bar{\Theta}$ *bilocal tomographic*

ii) maximal SSR $\quad V^{\sigma}_{\mathrm{AB}} = U(\sigma, \mathrm{A}, \mathrm{B}) \quad \Rightarrow \quad \bar{\Theta}$ *local tomographic*

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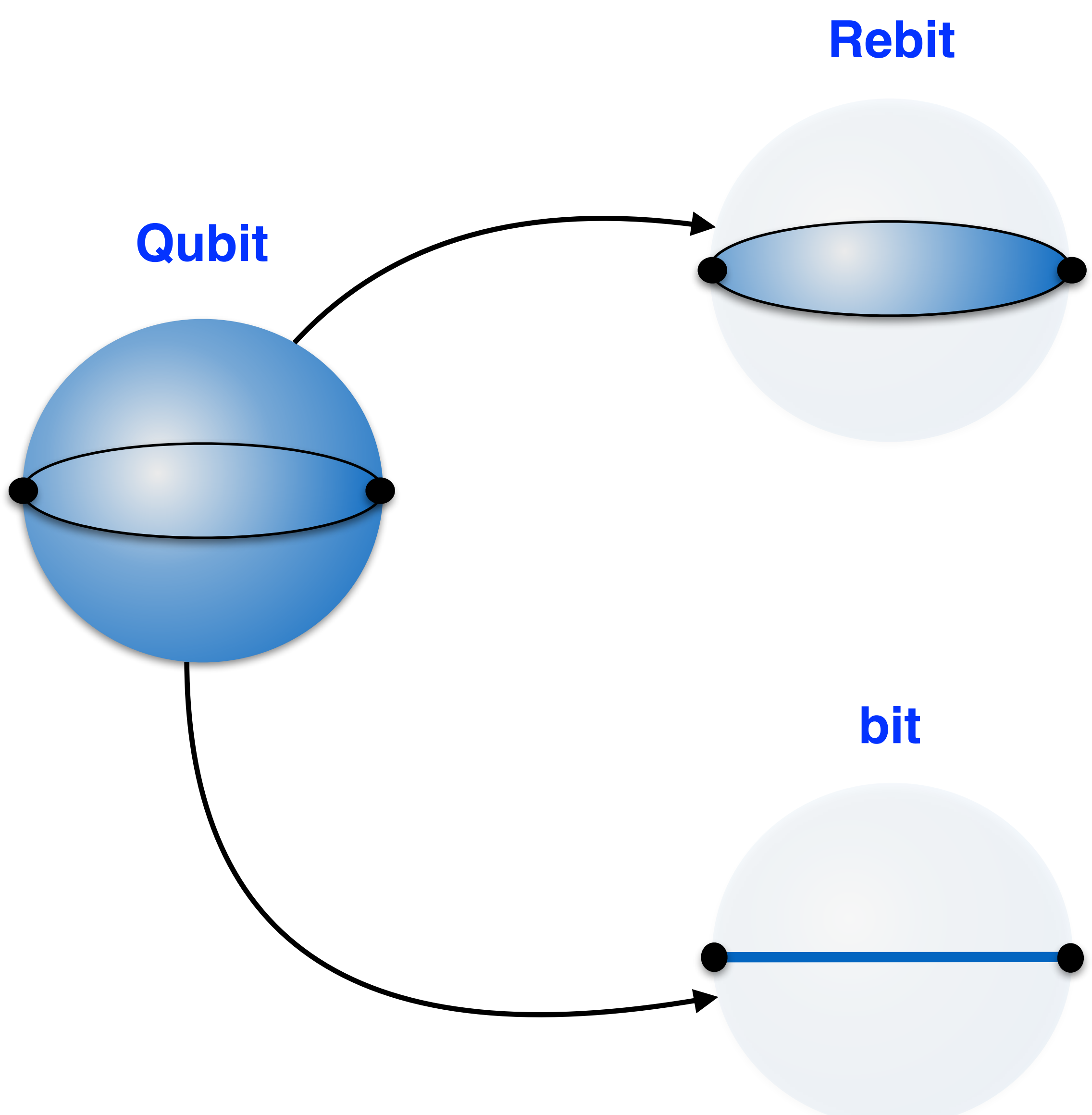
Fermionic and Real quantum theory



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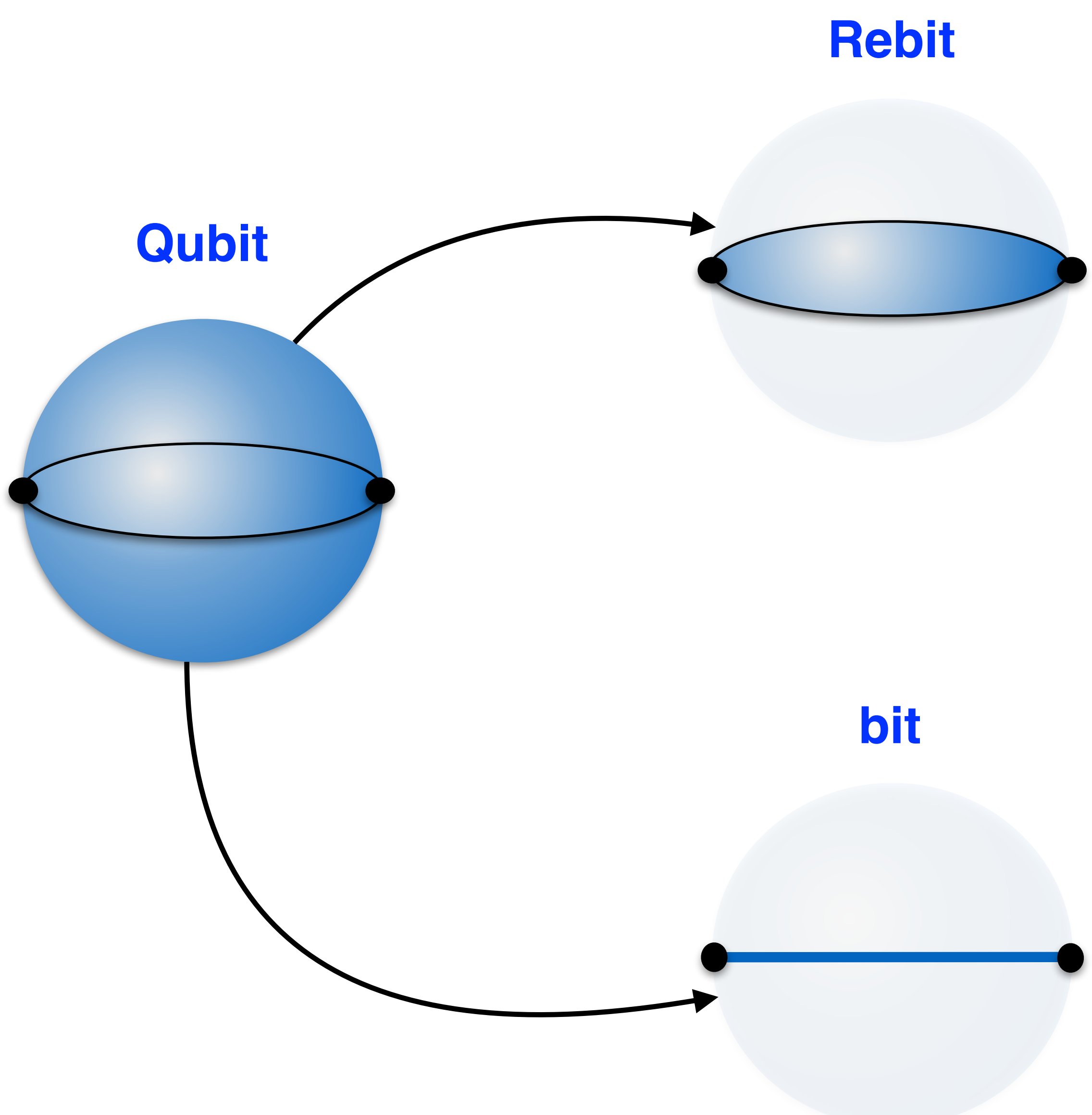
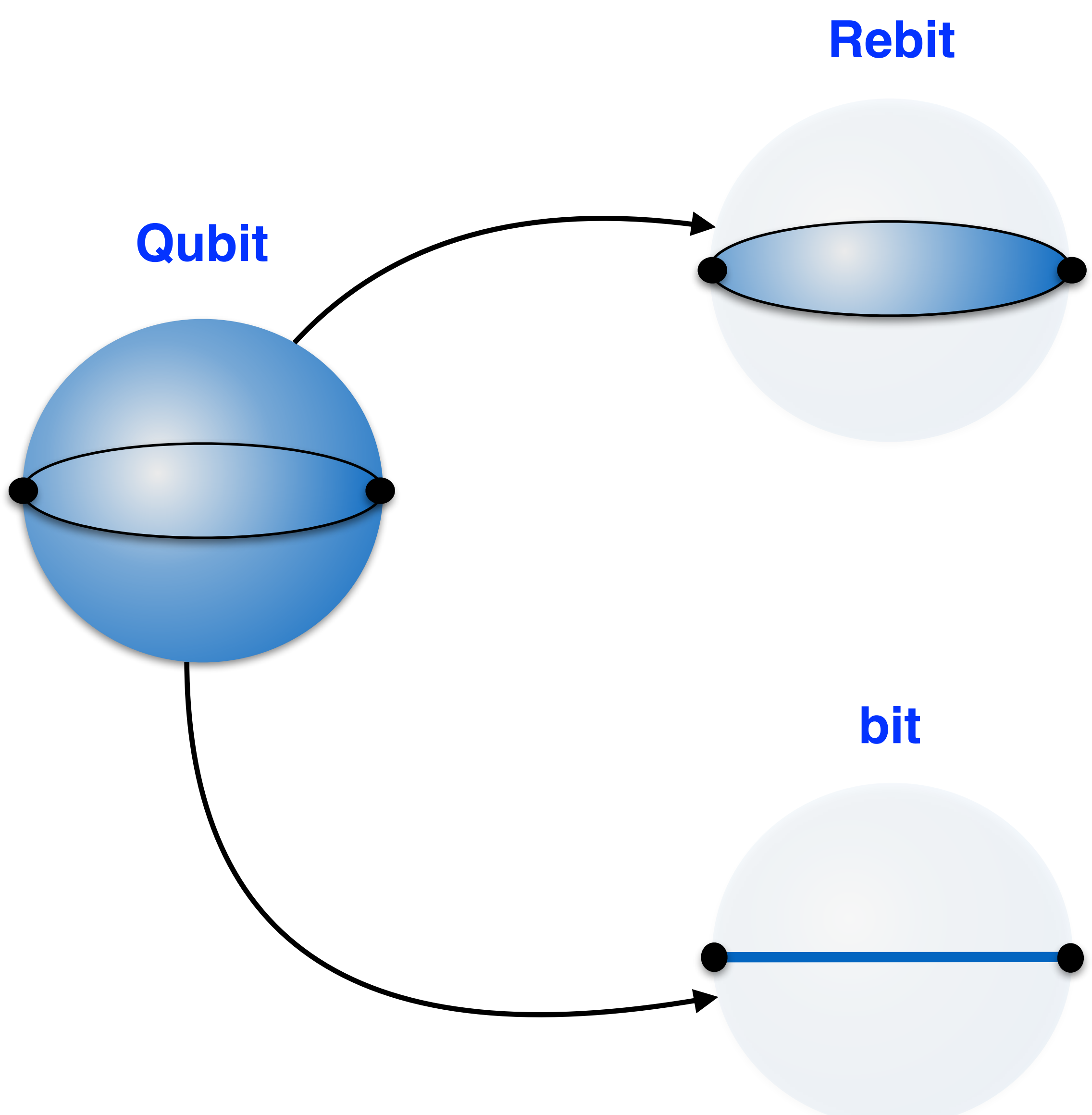


1 linear constraint

$$\mathrm{Tr}[\rho\sigma_y] = 0$$

Fermionic and Real quantum theory



1 linear constraint

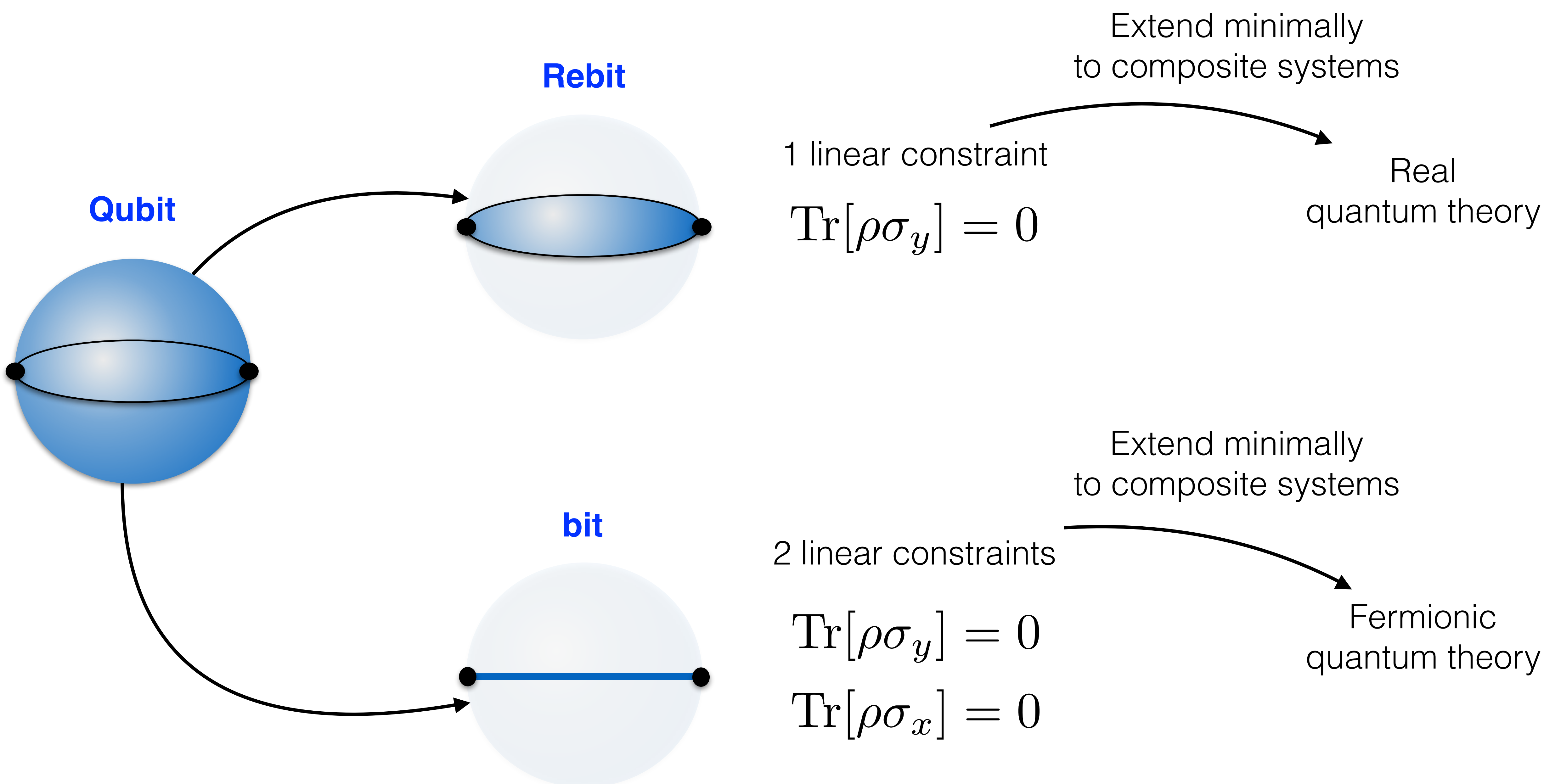
$$\mathrm{Tr}[\rho\sigma_y] = 0$$

2 linear constraints

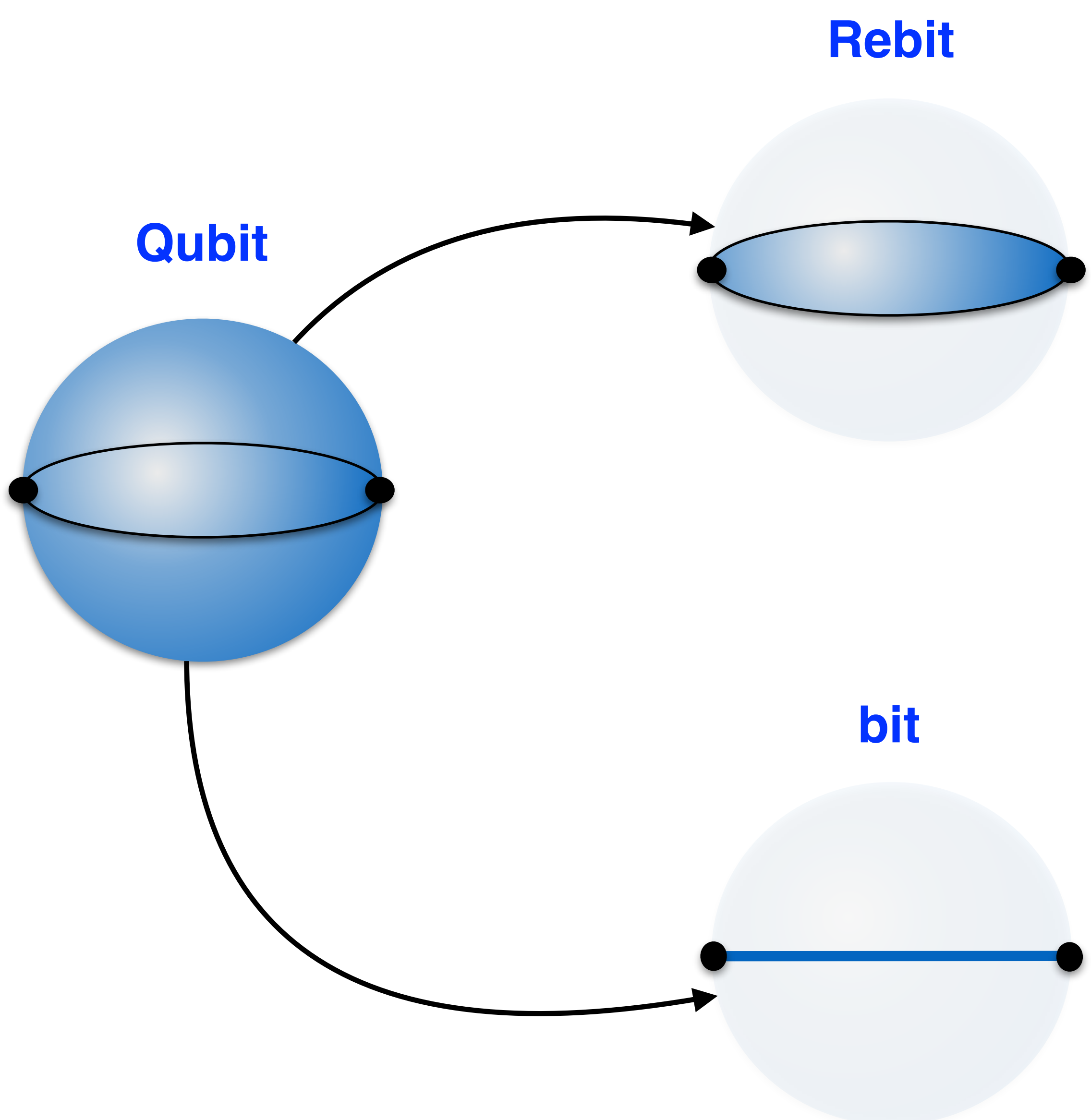
$$\mathrm{Tr}[\rho\sigma_y] = 0$$

$$\mathrm{Tr}[\rho\sigma_x] = 0$$

Fermionic and Real quantum theory



Fermionic and Real quantum theory



Extend minimally
to composite systems

1 linear constraint

$$\mathrm{Tr}[\rho\sigma_y] = 0$$

Real
quantum theory

Extend minimally
to composite systems

2 linear constraints

$$\mathrm{Tr}[\rho\sigma_y] = 0$$
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Fermionic
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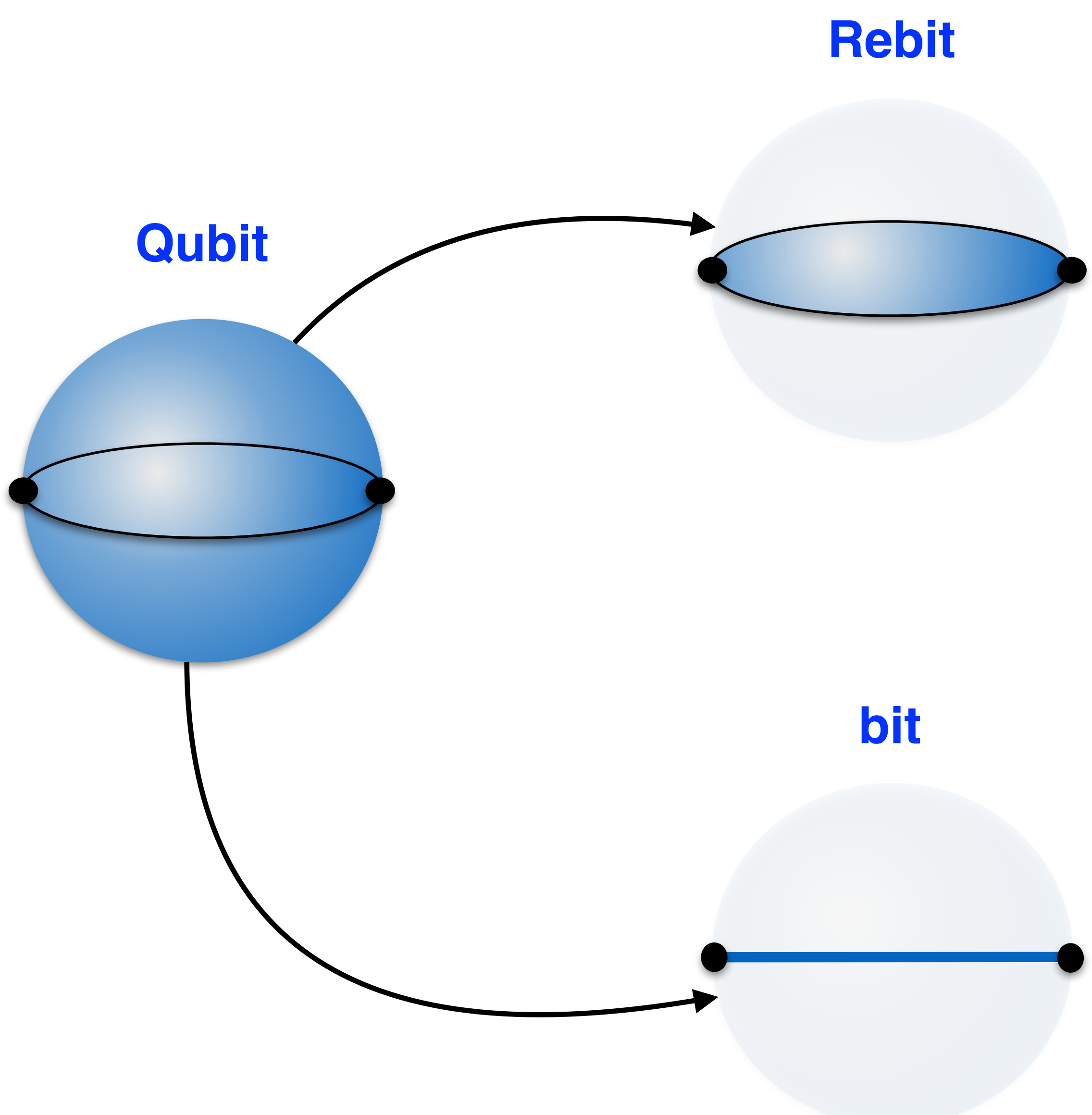
Real and Fermionic quantum theory are
(the only two)
minimal SSR of QT

Fermionic and Real quantum theory



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minimal SSR => bilocal tomography

G. M. D'Ariano, F. Manessi, P. Perinotti and A. Tosini, IJMPA (2014)
L. Hardy and W. K. Wootters, Foundations of Physics 42, 454 (2012)

5. Future perspectives

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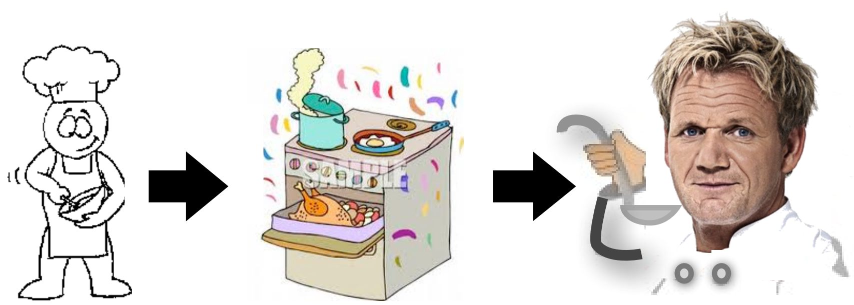

+ Axioms regarding how information can or cannot be manipulated

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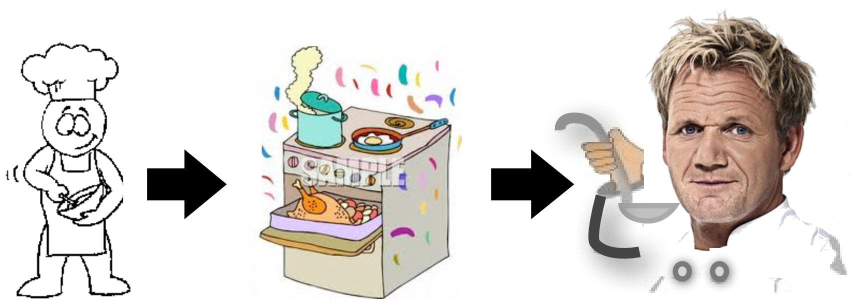

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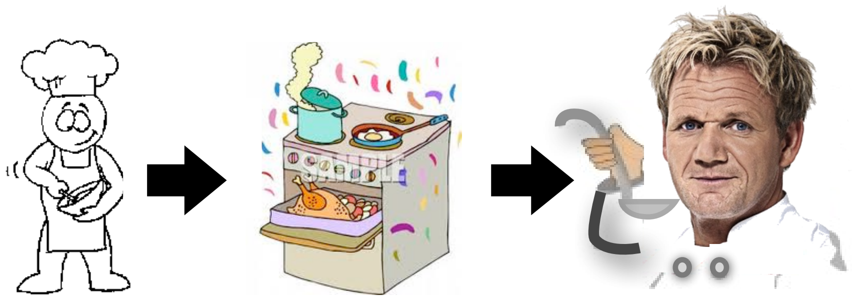

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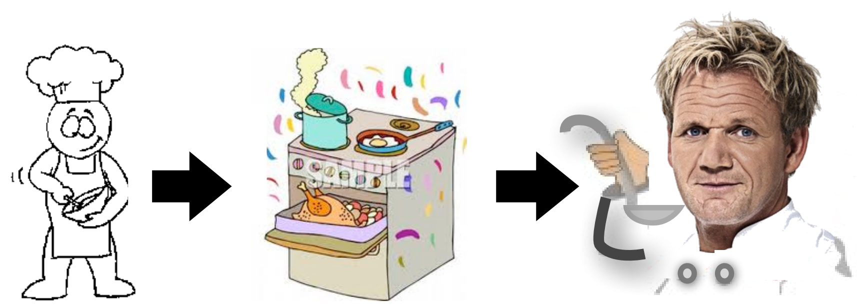

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Question: extend the operational informational framework to **Quantum Field Theory**

Alternative to **Algebraic Quantum Field theory**

Haag, R., Local quantum physics, volume 2, Springer Berlin, 1996.