Macroscopic non-contextuality as a principle for Almost Quantum Correlations

Joe Henson and Ana Belén Sainz

University of Bristol

University of BRISTOL

Characterising correlations

- Nonlocality:

  nontrivial communication complexity[1], no advantage for nonlocal computation[2], information causality[3], macroscopic locality[4], local orthogonality[5].

  Not enough[5,6] $\rightarrow$ Almost quantum correlations[6]

[1] van Dam, PhD thesis, University of Oxford (2000).
[2] Linden, Popescu, Short, Winter, arXiv:quant-ph/0610097.
[3] Pawlowski et al., Nature 461, 1101-1104 (2009).
[4] Navascués and Wunderlich, Proc. Roy. Soc. Lond. A 466:881-890 (2009).
[5] Fritz et al., Nat. Comm. 4, 2263 (2013).
[6] Navascués, Guryanova, Hoban and Acín, Nat. Comm. 6, 6288 (2015).
7
8

Characterising correlations

- Nonlocality:

  nontrivial communication complexity[1], no advantage for nonlocal computation[2], information causality[3], macroscopic locality[4], local orthogonality[5].

  Not enough[5,6] $\rightarrow$ Almost quantum correlations[6]

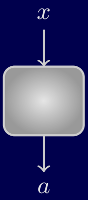
- Contextuality: Consistent exclusivity[7]

  Not enough[7], extra assumptions[8] $\rightarrow$ $\mathcal{Q}_1$

---

1 van Dam, PhD thesis, University of Oxford (2000).
2 Linden, Popescu, Short, Winter, arXiv:quant-ph/0610097.
3 Pawlowski et al., Nature 461, 1101-1104 (2009).
4 Navascués and Wunderlich, Proc. Roy. Soc. Lond. A 466:881-890 (2009).
5 Fritz et al., Nat. Comm. 4, 2263 (2013).
6 Navascués, Guryanova, Hoban and Acín, Nat. Comm. 6, 6288 (2015).
7 Acín, Fritz, Leverrier and Sainz, Comm. Math. Phys. 334(2), 533-628 (2015).
8 Amaral, Terra Cunha and Cabello, PRA 89, 030101 (2014) .

Contextuality scenarios



[9] A. Cabello, S. Severini and A. Winter, arXiv:1010.2163 (2010)

[10] A. Acín, T. Fritz, A. Leverrier, ABS arXiv:1210:4084 (2012).

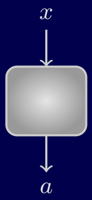
Contextuality scenarios

"Exclusivity" structure[9,10]

- Set of measurements
- Set of outcomes
- Identify outcomes of different measurements: same probability

[9] A. Cabello, S. Severini and A. Winter, arXiv:1010.2163 (2010)
[10] A. Acín, T. Fritz, A. Leverrier, ABS arXiv:1210:4084 (2012).

Contextuality scenarios



"Exclusivity" structure[9,10]

- Set of measurements
- Set of outcomes
- Identify outcomes of different measurements: same probability
  $\rightarrow$ measurements "share" outcomes

[9] A. Cabello, S. Severini and A. Winter, arXiv:1010.2163 (2010)
[10] A. Acín, T. Fritz, A. Leverrier, ABS arXiv:1210:4084 (2012).

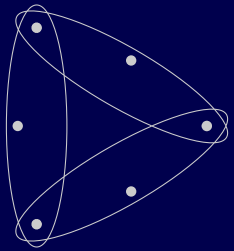
Contextuality scenarios

Hypergraphs:

- Vertices $\rightarrow$ events – measurement outcome

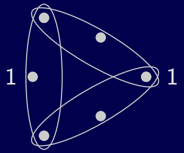
  $(a|x) \leftrightarrow v$

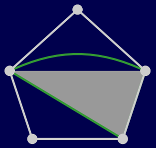
- Hyperedges $\rightarrow$ complete measurements – set of outcomes



Sets of allowed $p(v)$

Contextuality Scenarios





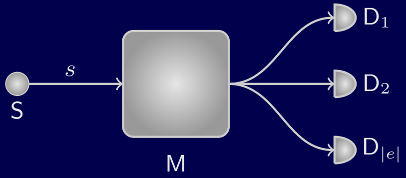
- Probabilistic Model: $\mathcal{G}(H)$
  $p : V \to [0, 1]$, properly normalised

- Classical models: $\mathcal{C}(H)$
  Determinism $\to$ convex combination of deterministic models

- Quantum models: $\mathcal{Q}(H)$
  $\exists \quad \mathcal{H}\,, \quad \rho\,, \quad \{P_v : v \in V\}, \sum_{v \in e} P_v = \mathbb{1}_{\mathcal{H}} \qquad \forall\, e \in E$
  $p\,(v) = \mathrm{tr}\,(\rho P_v)$

Macroscopic Non-Contextuality

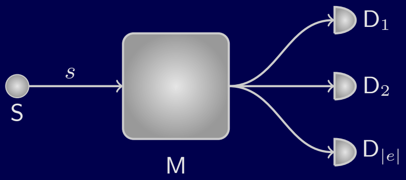
Micro scenario



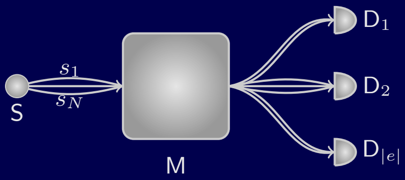
$p(v)$

Macroscopic Non-Contextuality

Micro scenario



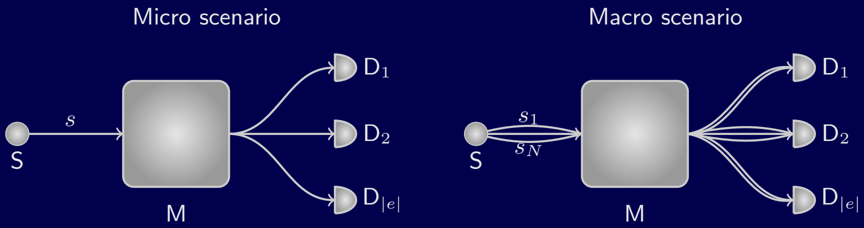
Macro scenario



$p(v)$

$\mathcal{P}_e\left(\{I^v\}_{v\in e}\right)$

Macroscopic Non-Contextuality



$p(v)$ $\mathcal{P}_e\left(\{I^v\}_{v\in e}\right)$

Macroscopic Non-Contextuality:

$p(v)$ satisfies MNC if $(N \to \infty)$ $\mathcal{P}_e\left(\{I^v\}_{v\in e}\right)$ is noncontextual

Macro and micro scenarios

$d_{i\,e}^{\,v} = 0, 1$ random variable $\quad \rightarrow \quad \bar{I}_e^v = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} (d_{i\,e}^{\,v} - p(v))$

Macro and micro scenarios

$d^{\,v}_{i\,e} = 0, 1$ random variable $\quad\rightarrow\quad \bar{I}^v_e = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} (d^{\,v}_{i\,e} - p(v))$

Constraints

- Normalisation: $\sum_{v \in e} \bar{I}^v_e = 0 \quad \forall\, e$

Macro and micro scenarios

$d_{i\,e}^{\,v} = 0, 1$ random variable $\quad \rightarrow \quad \bar{I}_e^v = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} (d_{i\,e}^{\,v} - p(v))$

Constraints

- Normalisation: $\sum_{v \in e} \bar{I}_e^v = 0 \quad \forall\, e$
- CLT: $N \rightarrow \infty$ distribution over $\bar{I}_e^v$ is Gaussian

  $\gamma_{uv}^e = \delta_{uv} p(v) - p(u)p(v)$

Macro and micro scenarios

$d^{\,v}_{i\,e} = 0, 1$ random variable $\quad \rightarrow \quad \bar{I}^v_e = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} (d^{\,v}_{i\,e} - p(v))$

Constraints

- Normalisation: $\sum_{v \in e} \bar{I}^v_e = 0 \quad \forall\, e$
- CLT: $N \rightarrow \infty$ distribution over $\bar{I}^v_e$ is Gaussian

  $\gamma^e_{uv} = \delta_{uv} p(v) - p(u)p(v)$
- MNC: $N \rightarrow \infty$ $\exists$ JPD $\mathcal{P}_{\mathsf{NC}}$ over the set of intensities for ALL outcomes.

  $\mathcal{P}_e(\{I^v\}_{v \in e}) = \int \left(\prod_{v \in V(H) \setminus e} dI^v \right) \mathcal{P}_{\mathsf{NC}}(\{I^v\}_{v \in V(H)})$

Macro and micro scenarios

$d^{\,v}_{i\,e} = 0, 1$ random variable $\quad\rightarrow\quad \bar{I}^v_e = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} (d^{\,v}_{i\,e} - p(v))$

Constraints

- Normalisation: $\sum_{v\in e} \bar{I}^v_e = 0 \quad \forall\, e$
- CLT: $N \rightarrow \infty$ distribution over $\bar{I}^v_e$ is Gaussian

  $\gamma^e_{uv} = \delta_{uv} p(v) - p(u)p(v)$
- MNC: $N \rightarrow \infty$ $\exists$ JPD $\mathcal{P}_{\mathsf{NC}}$ over the set of intensities for ALL outcomes.

  $\mathcal{P}_e(\{I^v\}_{v\in e}) = \int \left(\prod_{v\in V(H)\setminus e} dI^v\right) \mathcal{P}_{\mathsf{NC}}(\{I^v\}_{v\in V(H)})$

  $\gamma_{uv} := \langle \bar{I}^u\, \bar{I}^v \rangle \rightarrow \sum_{u\in e} \gamma_{uv} = 0,\ \gamma \geq 0.$

Macroscopic non-contextuality and $\mathcal{Q}_1$

Macroscopic non-contextuality

$p \in \mathcal{G}(H)$ is MNC if $\exists\ \gamma \geq 0$ such that:

- $\sum_{u \in e} \gamma_{uv} = 0$;
- $(u, v \in e$ and $u \neq v) \Rightarrow \gamma_{uv} = -p(u)p(v)$;
- $\gamma_{vv} = p(v) - p(v)^2$.

Macroscopic non-contextuality and $\mathcal{Q}_1$

Macroscopic non-contextuality

$p \in \mathcal{G}(H)$ is MNC if $\exists\ \gamma \geq 0$ such that:

- $\sum_{u \in e} \gamma_{uv} = 0$;
- $(u, v \in e \text{ and } u \neq v) \Rightarrow \gamma_{uv} = -p(u)p(v)$;
- $\gamma_{vv} = p(v) - p(v)^2$.

$\mathcal{Q}_1$ models

$p \in \mathcal{G}(H)$ is a $\mathcal{Q}_1$ model if $\exists\ M \geq 0$ such that:

- $\sum_{u \in e} M_{uv} = p(v)$ for all $u \in V(H)$;
- $(u, v \in e \text{ and } u \neq v) \Rightarrow M_{uv} = 0$;
- $M_{vv} = P(v)$;
- $M_{1v} = P(v)$ and $M_{11} = 1$.

Macroscopic non-contextuality and $\mathcal{Q}_1$

Macroscopic non-contextuality

$p \in \mathcal{G}(H)$ is MNC if $\exists\ \gamma \geq 0$ such that:

- $\sum_{u \in e} \gamma_{uv} = 0$;
- $(u, v \in e$ and $u \neq v) \Rightarrow \gamma_{uv} = -p(u)p(v)$;
- $\gamma_{vv} = p(v) - p(v)^2$.

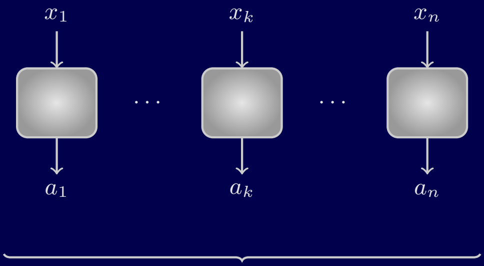
$\mathcal{Q}_1$ models

$p \in \mathcal{G}(H)$ is a $\mathcal{Q}_1$ model if $\exists\ M \geq 0$ such that:

- $\sum_{u \in e} M_{uv} = p(v)$ for all $u \in V(H)$;
- $(u, v \in e$ and $u \neq v) \Rightarrow M_{uv} = 0$;
- $M_{vv} = P(v)$;
- $M_{1v} = P(v)$ and $M_{11} = 1$.

$$\gamma_{uv} = M_{uv} - p(u)p(v) \quad \longrightarrow \quad p \text{ is MNC iff } p \in \mathcal{Q}_1(H)$$

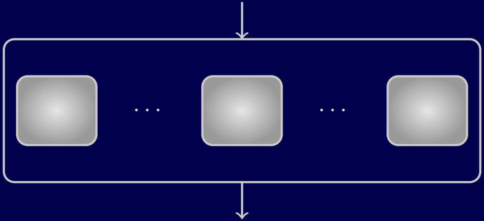
Bell scenarios as contextuality scenarios



$$P\left(a_1 \dots a_n | x_1 \dots x_n\right)$$

Bell scenarios as contextuality scenarios

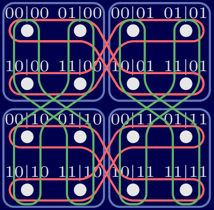
$$x = (x_1, \ldots, x_n)$$



$$a = (a_1, \ldots, a_n)$$

$$P(a_1 \ldots a_n | x_1 \ldots x_n) \quad \rightarrow \quad P(a|x)$$

Bell Scenarios as contextuality scenarios



$\mathcal{B}_{2,2,2}$

$H = \mathcal{B}_{n,m,d}$:

Vertices – events:
$\{(a_1 \ \dots \ a_n | x_1 \ \dots \ x_n)\}_{a_1 \dots a_n, x_1 \dots x_n}$

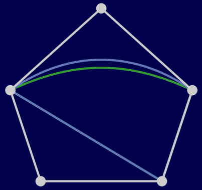
Hyperedges: correlated measurements

$\mathcal{G}(\mathcal{B}_{n,m,d}) = \mathcal{NS}(n, m, d)$

Bell Scenarios

AFLS[11]: p is almost quantum[12] in (n, m, d) iff $p \in \mathcal{Q}_1(\mathcal{B}_{n,m,d})$

(n, m, d): p is Almost quantum iff p is MNC in $\mathcal{B}_{n,m,d}$



[11] Acín, Fritz, Leverrier and Sainz, Comm. Math. Phys. 334(2), 533-628 (2015).
[12] Navascués, Guryanova, Hoban and Acín, Nat. Comm. 6, 6288 (2015).

Conclusions and open problems

- Generalise ML to contextuality scenarios
- Strengthen ML in Bell scenarios
  $\rightarrow$ correlated measurements
- MNC fully characterises almost quantum models without extra assumptions (as opposed to CE)
- MNC directly applies to multipartite Bell scenarios (as opposed to CE)
- First characterisation of almost quantum correlations from a physical principle.

Conclusions and open problems

- Generalise ML to contextuality scenarios
- Strengthen ML in Bell scenarios
  $\rightarrow$ correlated measurements
- MNC fully characterises almost quantum models without extra assumptions (as opposed to CE)
- MNC directly applies to multipartite Bell scenarios (as opposed to CE)
- First characterisation of almost quantum correlations from a physical principle.
- From Almost quantum to quantum
  $\rightarrow$ sequences of measurements?

Thanks !!!

Joe Henson and **ABS** – PRA 91, 042114 (2015). (arXiv:1501.06052)