# Inferring Causal Structure: a Quantum Advantage

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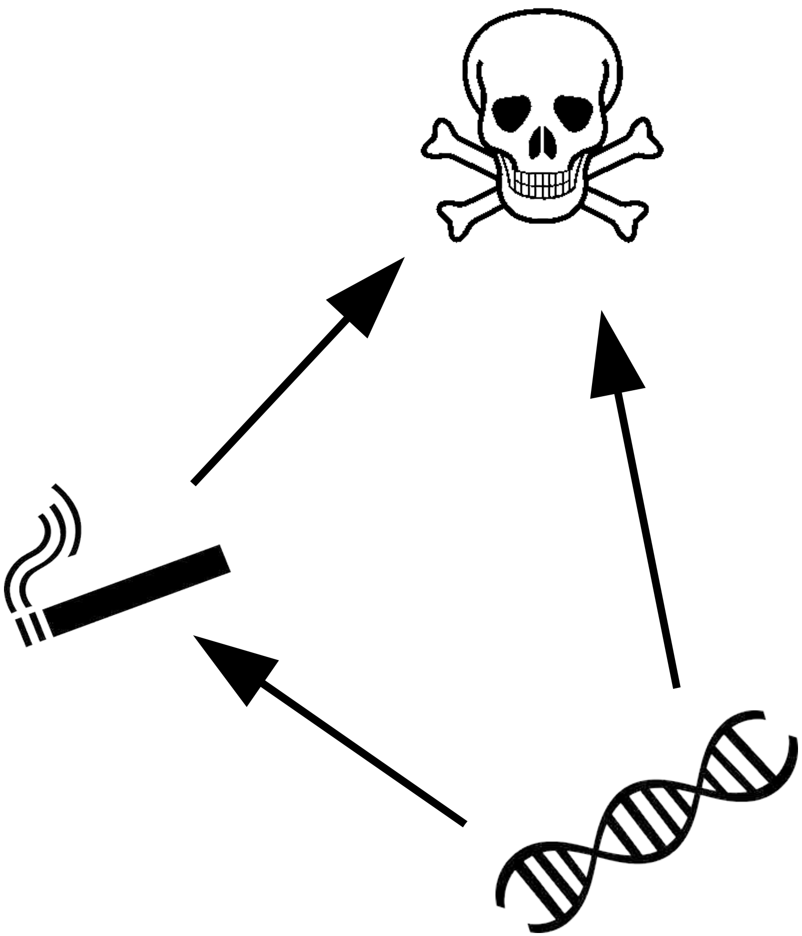
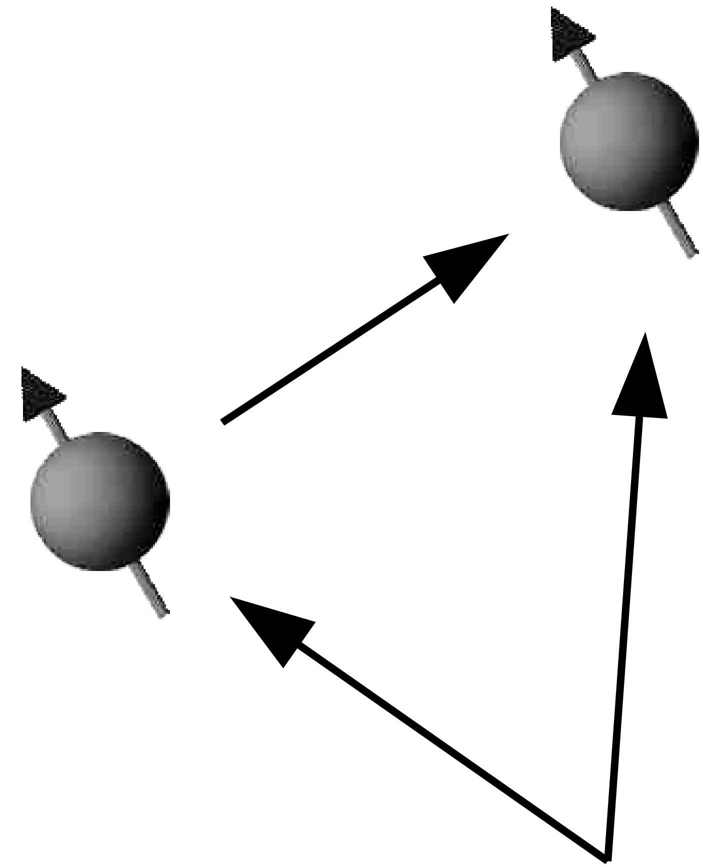
In a nutshell:

# Quantum correlations *can* imply causation



observed correlations compatible with both causal relations



observed correlations can herald causal relation

# Outline

1. Why causal explanations?
2. The task: causal inference – and why it is hard
3. Quantum causal inference
4. The quantum advantage
5. Experimental realization
6. Applications to open system dynamics
7. Outlook: superpositions of causal structures

# 1. Why causal explanations?

# Clinical trial of [redacted]

**Introduction**

Rather than merely observing correlations between events, science seeks to explain these correlations in terms of causal influences. In the context of classical variables, the concept of causation has been rigorously defined, and a framework for describing systems in terms of their causal relations has been established [Pearl_book, SpirtesEtAl_book].

**Method**

Its applications are manifold; a testament to the fact that a causal model captures the essence of “how the system works”. In a sense, it describes how information flows from one event to the other. What would a similar account of the relations between a set of quantum variables look like? I will discuss some ways in which classical causal models must be adapted to accommodate quantum variables, highlighting how causation and information processing are different from the classical case.

**Results**

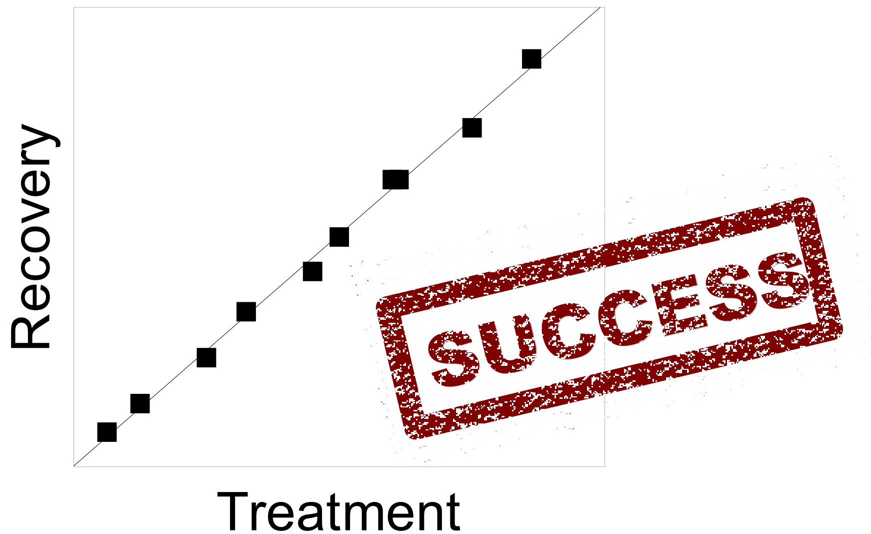
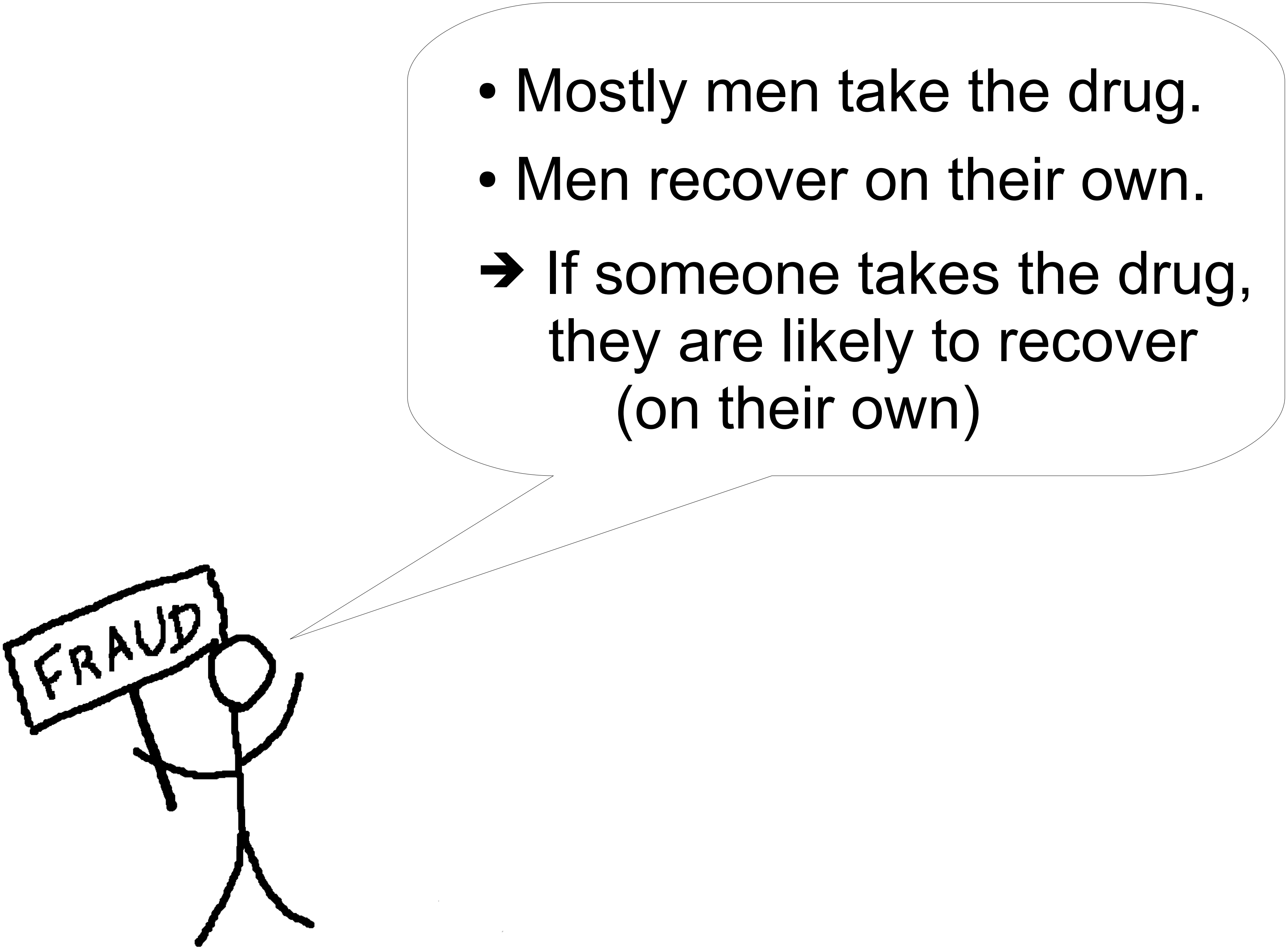


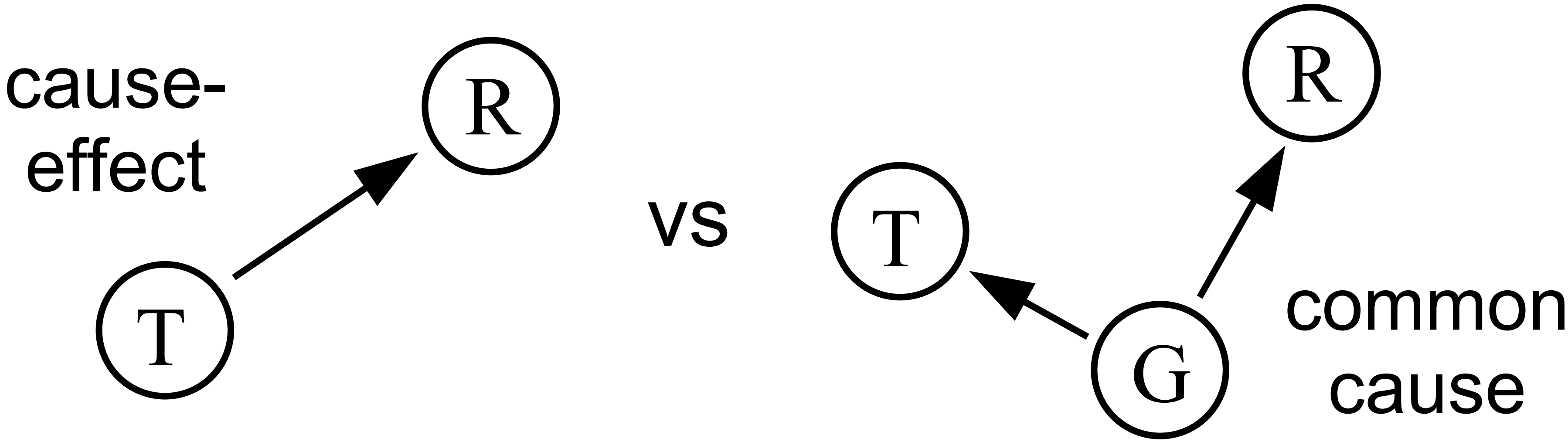

Fig. 1: Recovery correlates with treatment to a statistical significance of 20 standard deviations.

**Conclusion**

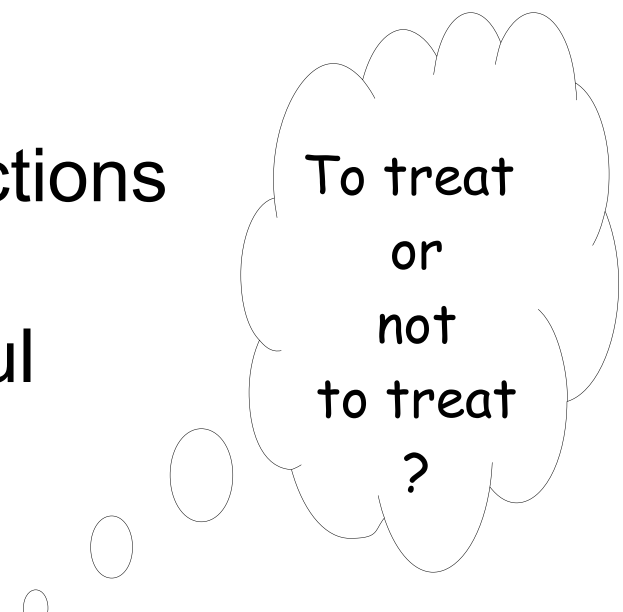
In particular, one such difference allows us to solve a task that is impossible to solve classically. “Causal inference” refers to the problem of determining the causal relations between a set of variables, given observational data. In the case of two classical variables, the correlations that can arise if one variable is a direct cause of the other are precisely the same as those that can arise from a common cause acting on both, so it is impossible to deduce the causal structure from them. Yet for quantum variables, we show that the correlations do encode a signature of the causal structure, which allows us to solve the causal inference problem. We illustrate this with data from a proof-of-concept experiment that corroborates our scheme for quantum causal inference [Agnew_draft].


• Mostly men take the drug.
• Men recover on their own.
➔ If someone takes the drug, they are likely to recover (on their own)
FRAUD

# More than correlation: Causation



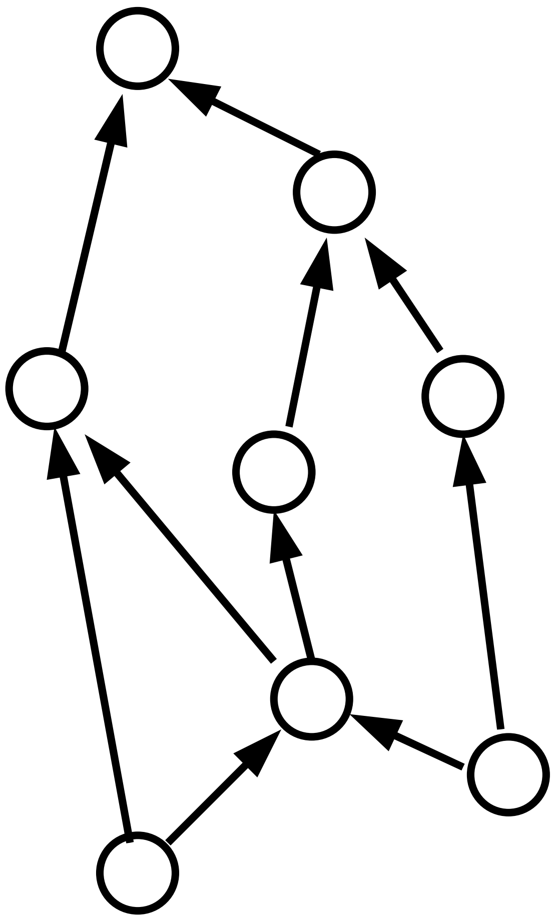

- "how things work"
- independent mechanisms allow predictions under changing circumstances
- causal models proved extremely useful

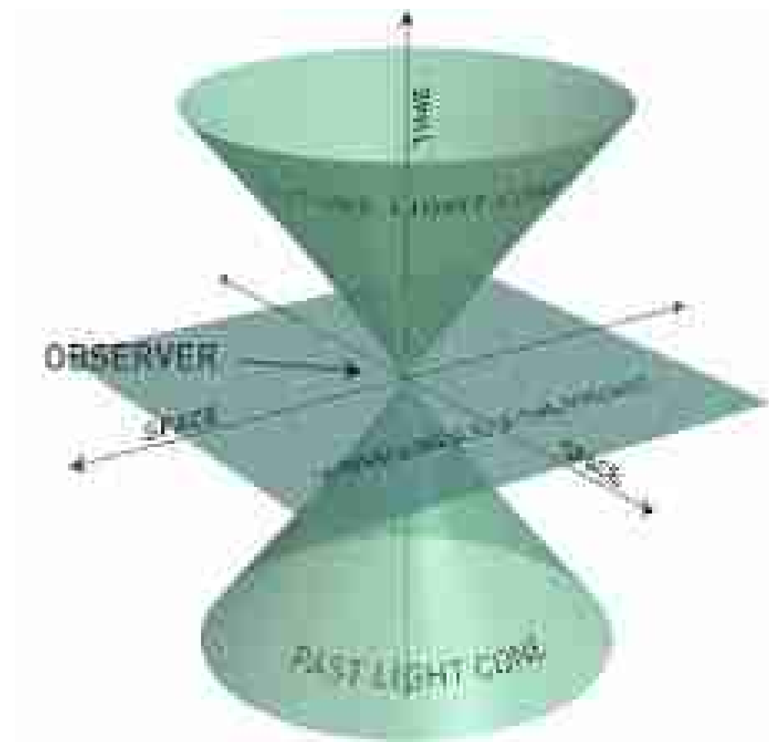




"Causality – reasoning, models and inference", J. Pearl, Cambridge University Press, 2009.
"Causation, Prediction, and Search", Spirtes, Glymour and Scheines, MIT Press, 2000.

# Causality and quantum foundations

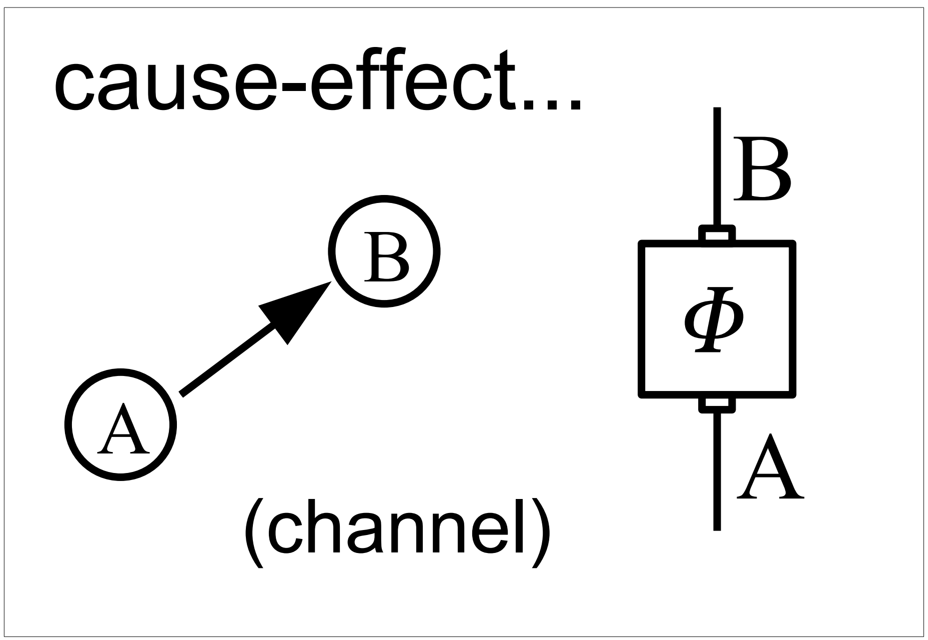


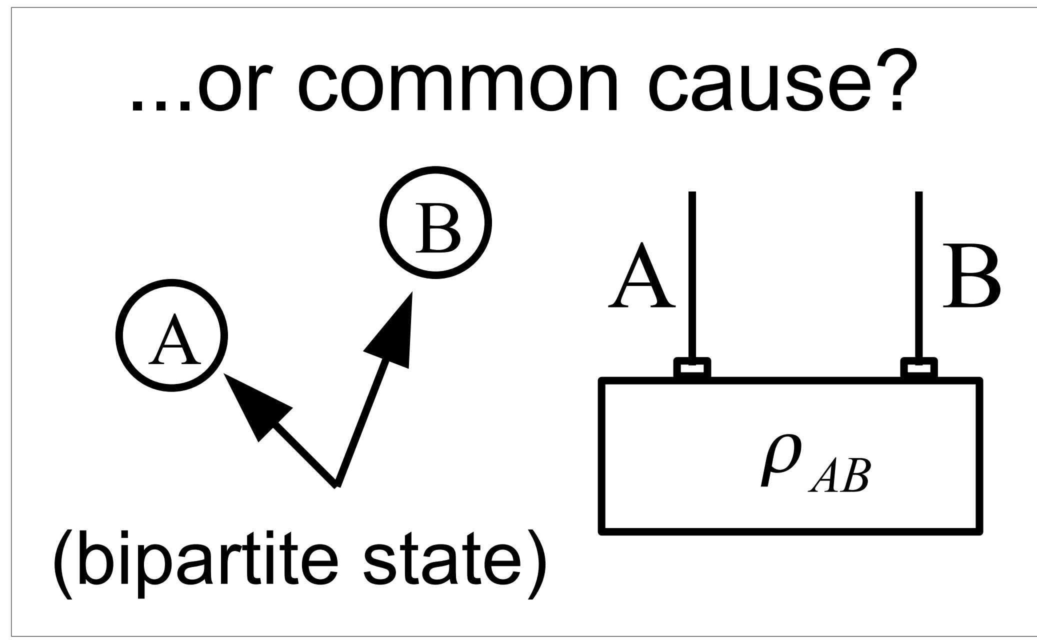


# 2. The task: causal inference – and why it is hard

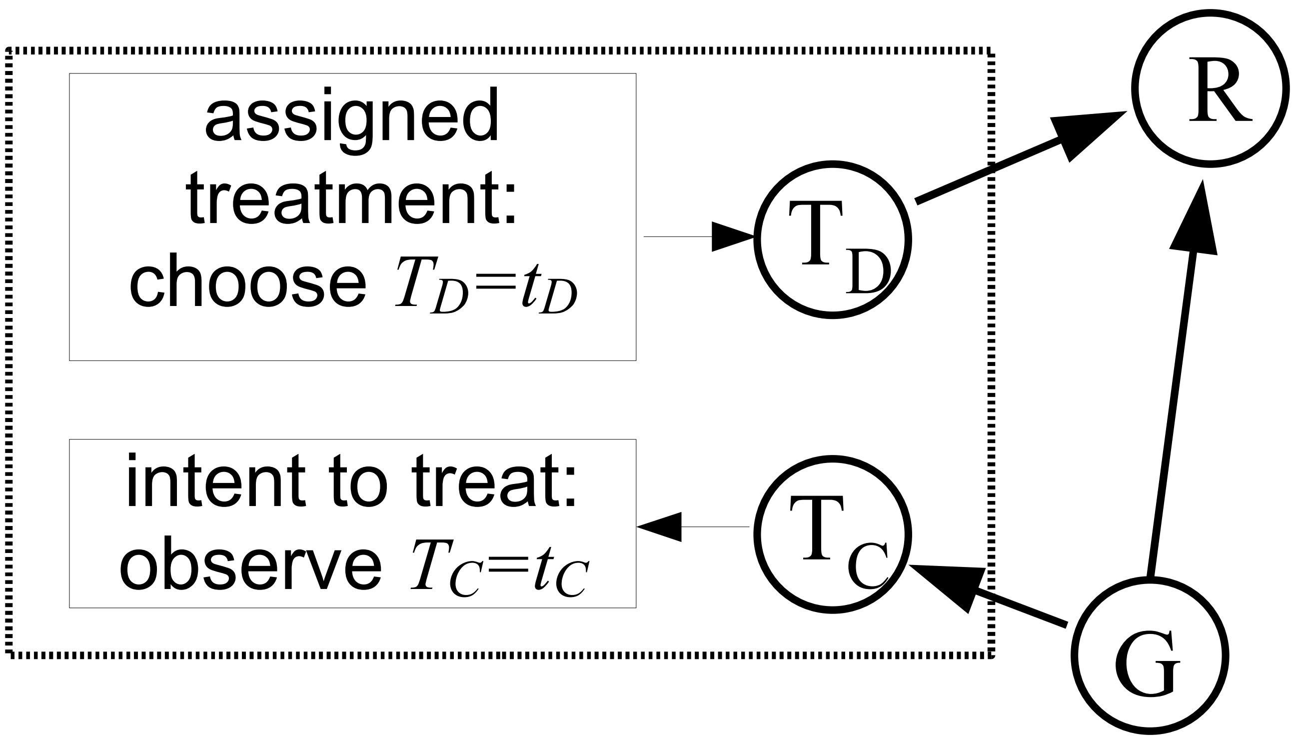
# Inferring causal structure

Given statistics *P(A,B)* for two variables, ...



# Randomized drug trials: when causal inference is easy

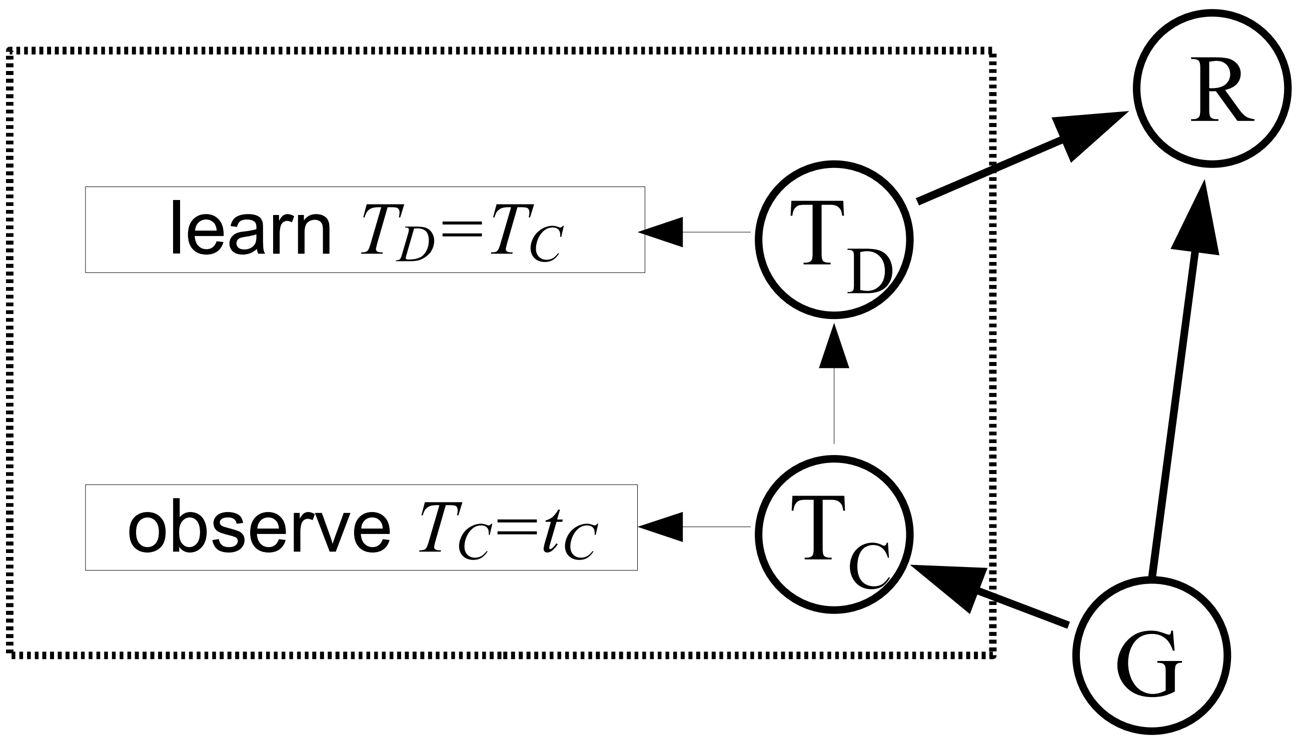



$$Corr(R, T_D) \Rightarrow$$ cause-effect

$$Corr(R, T_C) \Rightarrow$$ common cause

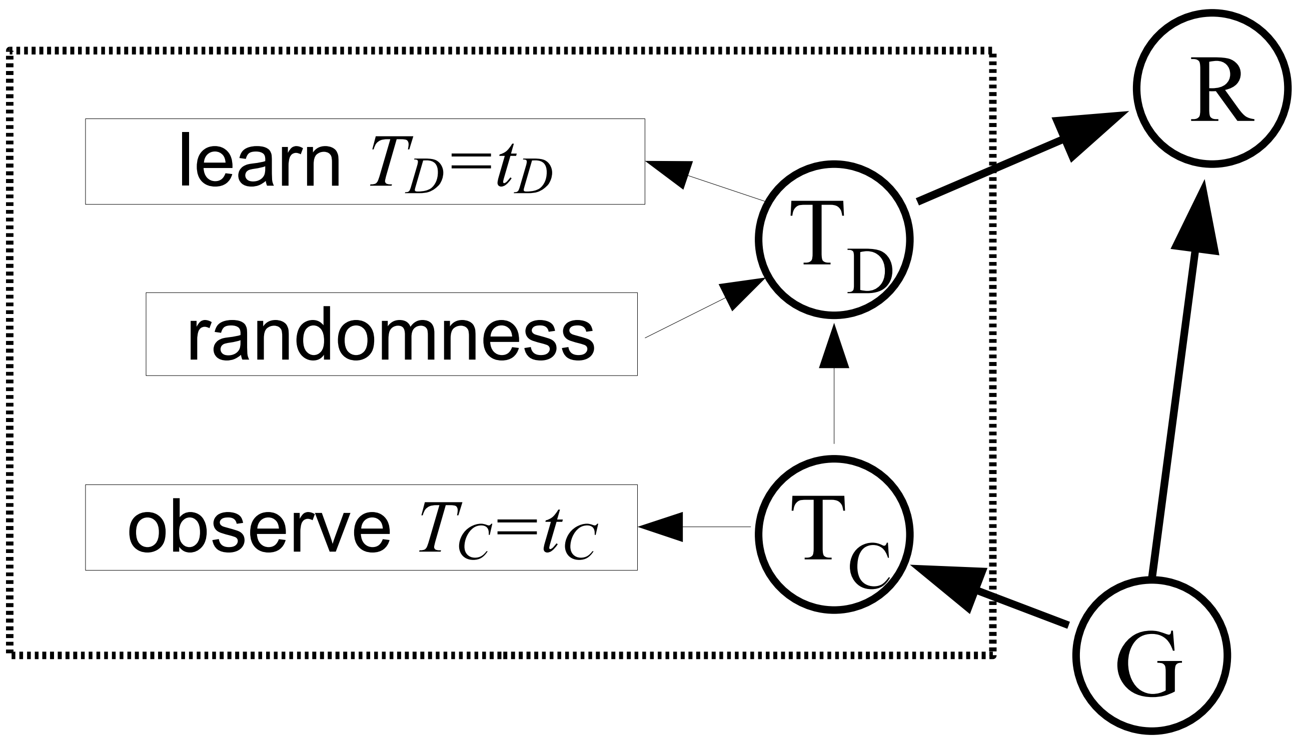
$\Rightarrow$ Causal inference becomes *trivial*.

No randomization




$Corr(R, T_D) = Corr(R, T_C) \quad \Rightarrow$ Causal inference becomes *impossible*.

# What makes causal inference possible?



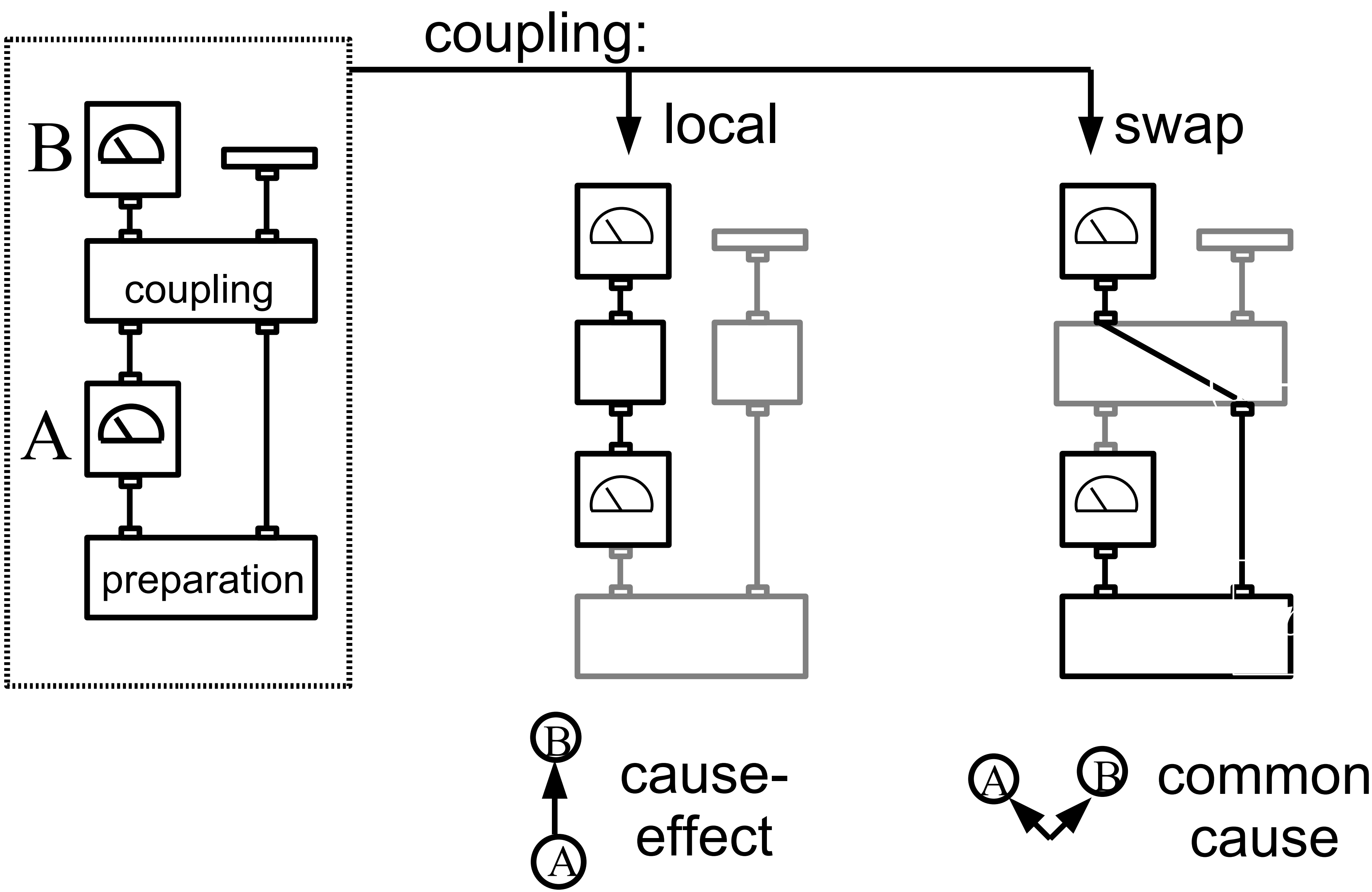

"information asymmetry":

independent information about $T_C$ and $T_D$

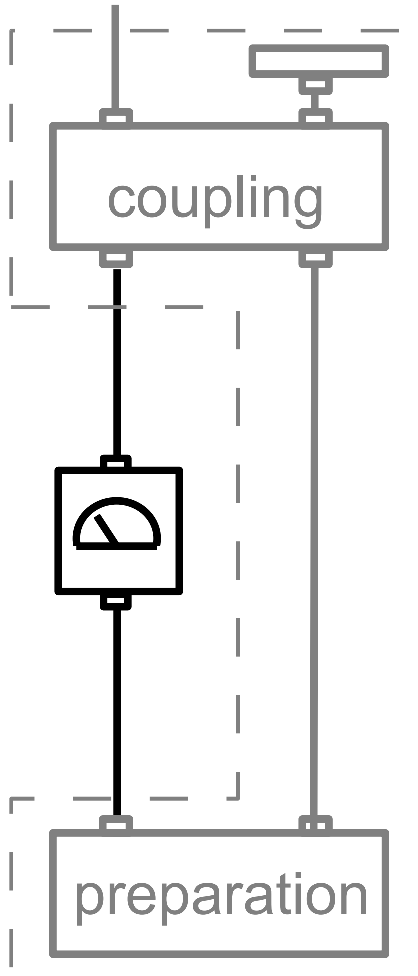
$\Rightarrow$ correlations with $R$ reveal causal structure

# 3. Quantum causal inference

# Two quantum variables with tunable causal relation

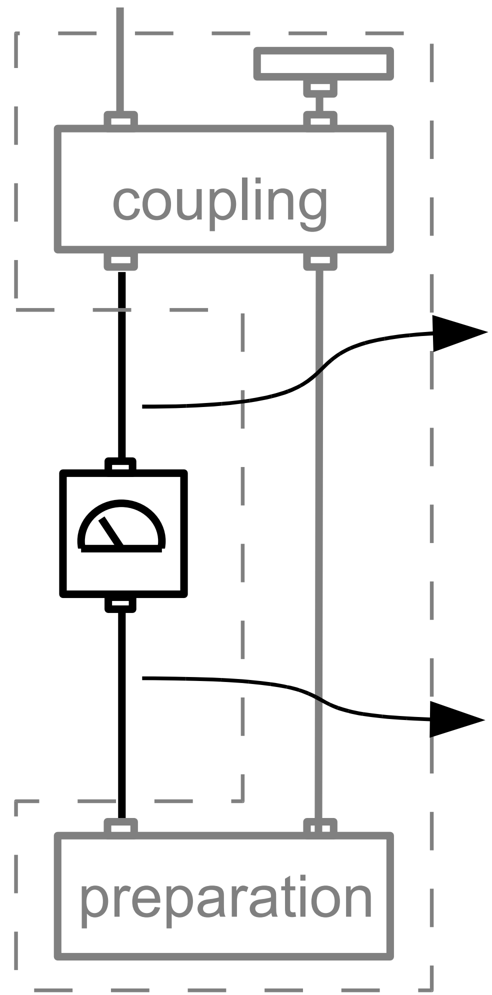


# Information symmetry for quantum systems




- projective measurement $|\psi\rangle$
- no prior information $\frac{1}{d}\mathbf{1}$

# Information symmetry for quantum systems




learn about system after measurement: $|\psi\rangle$

- projective measurement $|\psi\rangle$

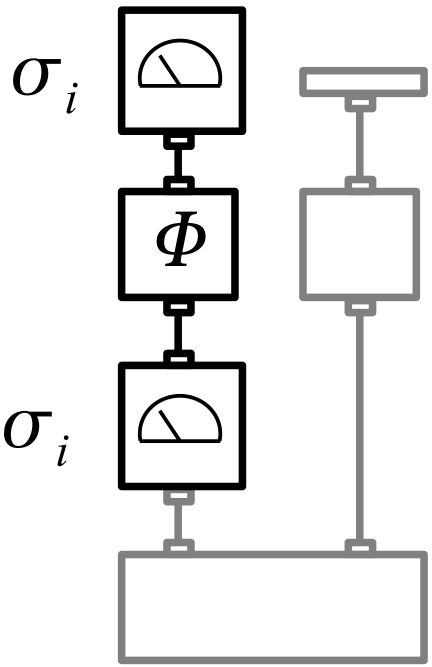
learn about system before measurement: $|\psi\rangle$

- no prior information $\frac{1}{d}\mathbf{1}$

# 4. The quantum advantage

How observed correlations
can reflect the causal relation

# Intuitive example

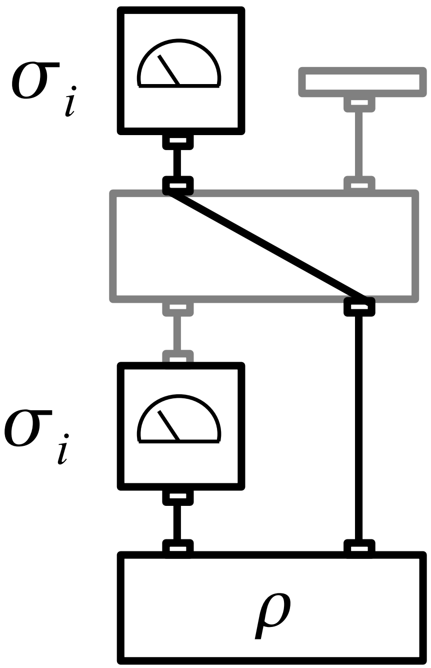



| $\Phi$ | $C_{xx}$ | $C_{yy}$ | $C_{zz}$ |
|---|---|---|---|
| id | +1 | +1 | +1 |
| X | +1 | -1 | -1 |
| Y | -1 | +1 | -1 |
| Z | -1 | -1 | +1 |

- channel $\Phi$
- measure $\sigma_i \otimes \sigma_i$
- correlation or anti-correlation?

$\Rightarrow$ proper rotations of Bloch sphere

# Intuitive example




| $\rho$ | $C_{xx}$ | $C_{yy}$ | $C_{zz}$ |
|---|---|---|---|
| $\Psi^-$ | -1 | -1 | -1 |
| $\Phi^-$ | -1 | +1 | +1 |
| $\Phi^+$ | +1 | -1 | +1 |
| $\Psi^+$ | +1 | +1 | -1 |

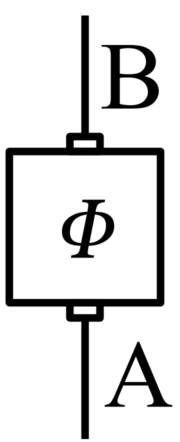
- bipartite state $\rho$
- measure $\sigma_i \otimes \sigma_i$
- correlation or anti-correlation?

$\Rightarrow$ improper rotations of Bloch sphere

## Choi-Jamiołkowski isomorphism

between channels and operators: $\Phi\left(\rho_A\right)=\rho_B=Tr_A(\chi_{AB}\rho_A\otimes\mathbf{1}_{\boldsymbol{B}})$

### Cause-effect




channel:

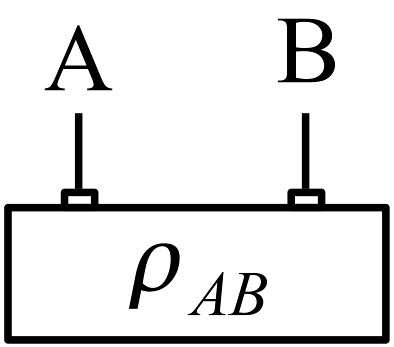
$\Phi^{ce}$ is CP

operator:

$T_A\left(\chi_{AB}^{ce}\right)$ is Pos

$\chi_{AB}^{ce}$ is PPT

### Common-cause




channel:

$\Phi^{cc}\circ T_A$ is CP

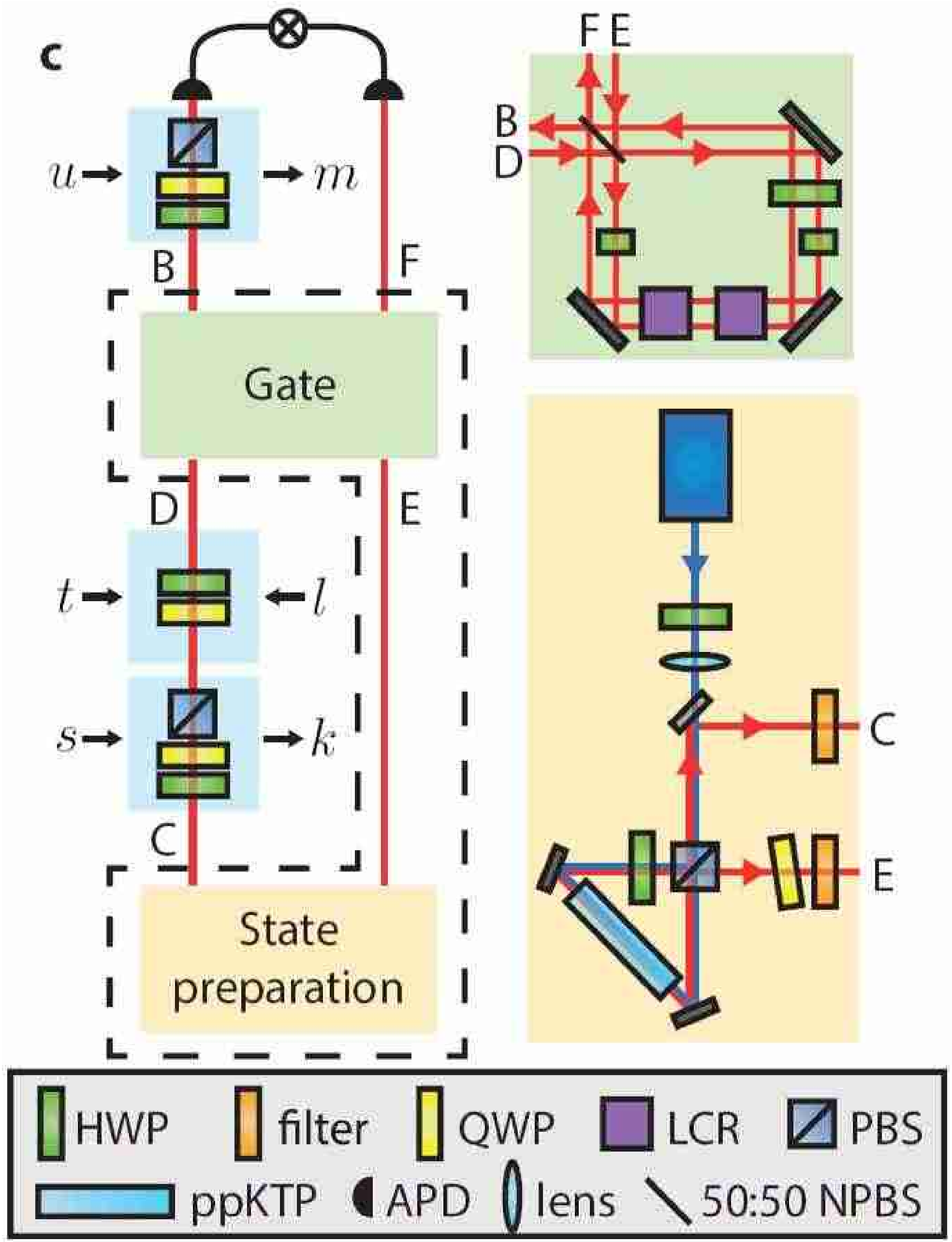
$\Phi^{cc}$ is cCP

$\chi_{AB}^{cc}\equiv\rho_A^{-1/2}\rho_{AB}\rho_A^{-1/2}$

operator:

$\chi_{AB}^{cc}$ is Pos

# 5. Experimental realization




coupling:

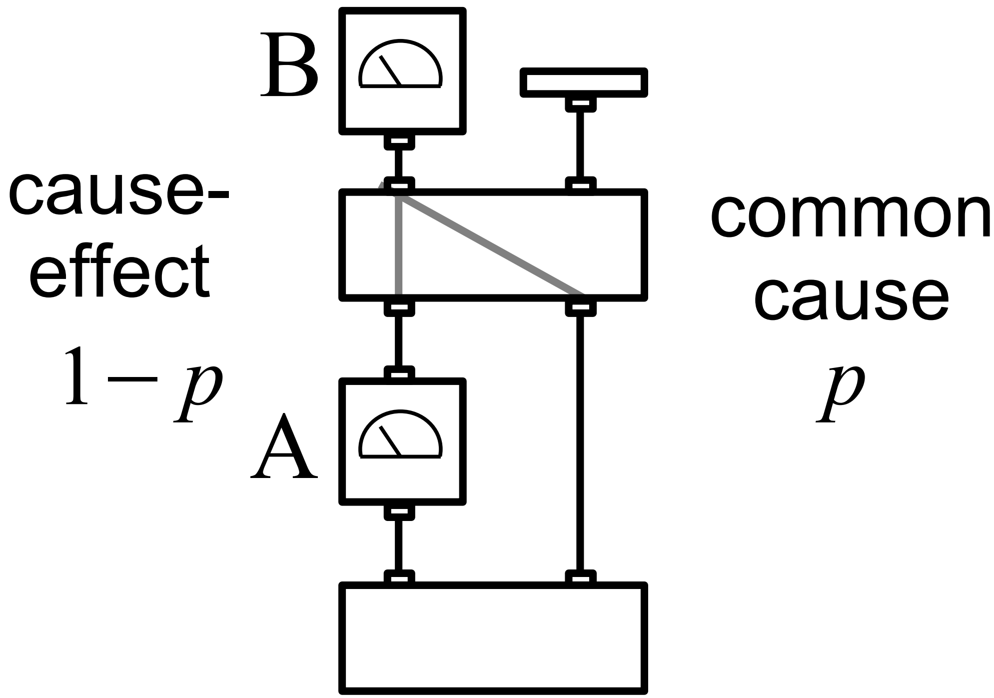
$$(1-p)\mathbf{1}+p\,swap$$

interferometer
with LCRs

preparation:
downconversion gives pairs of polarization-entangled photons

Resch group, Institute for Quantum Computing, Waterloo, Canada

# Resolving a probabilistic mixture



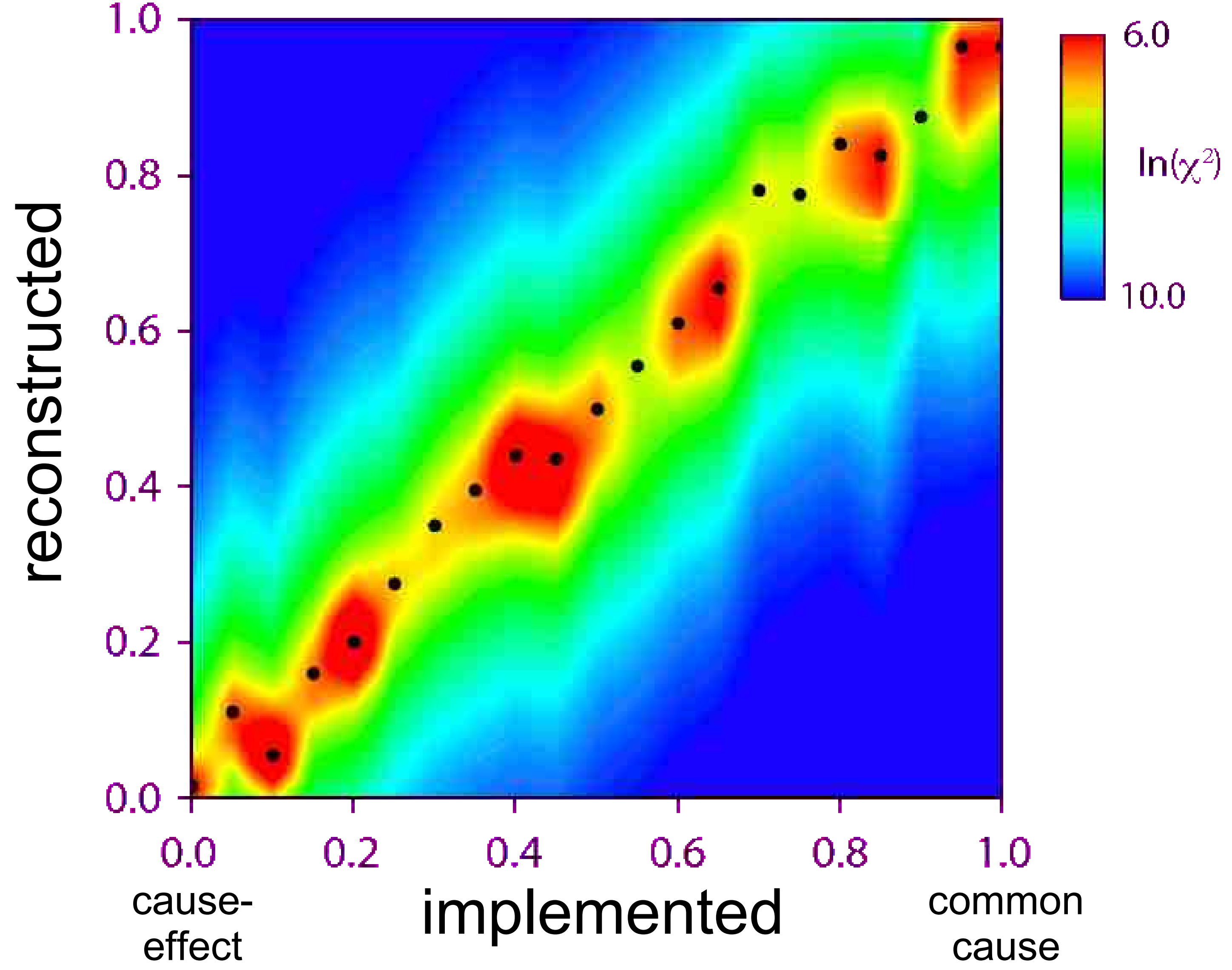

- implement $p$
- collect data
- fit to

  $$(1-p)\Phi^{ce} + p\,\Phi^{cc}$$

  (minimize residue $\chi^2$)

  ⇒ reconstruct $p$

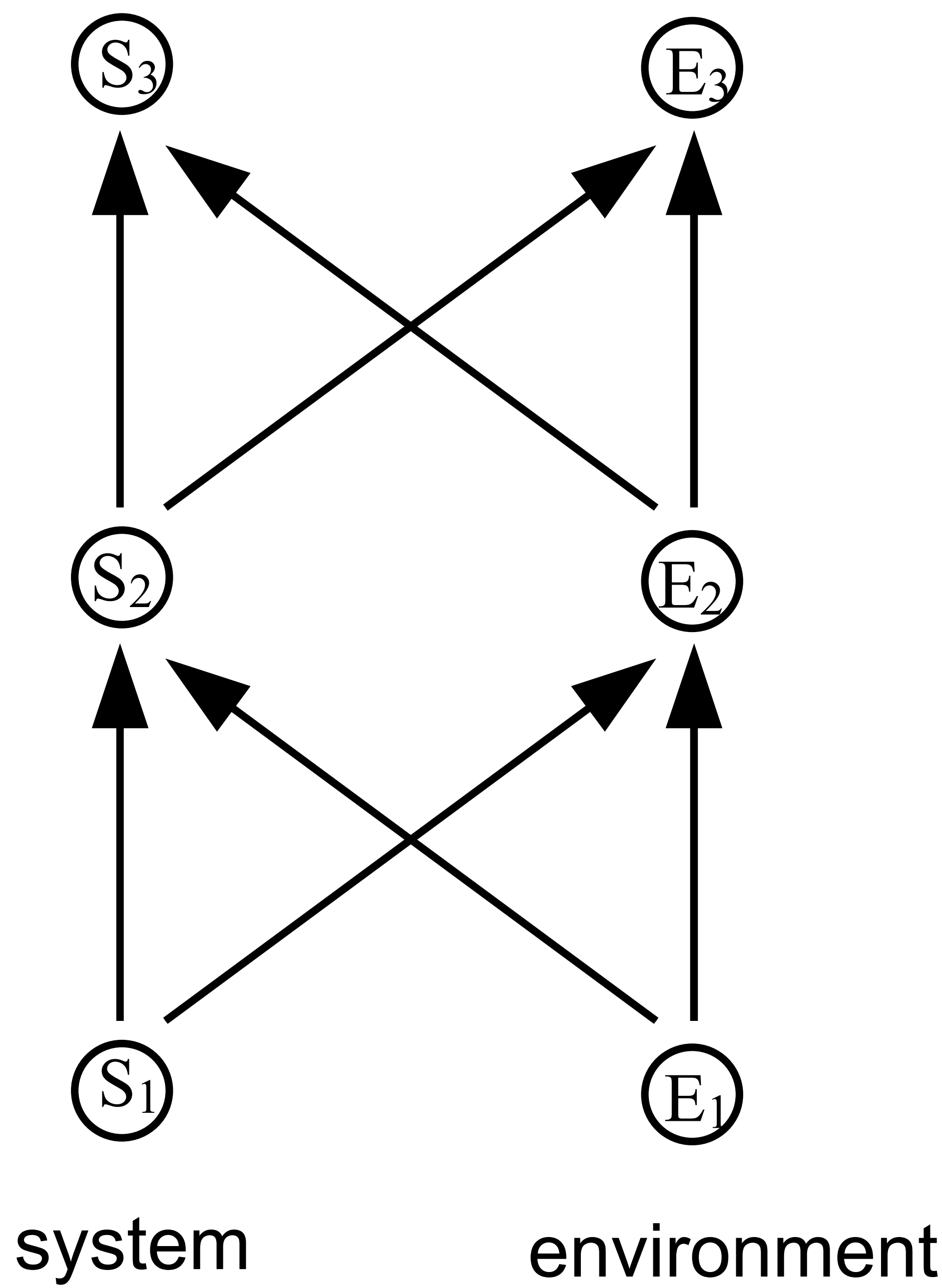
# Probability of common cause – experimental results



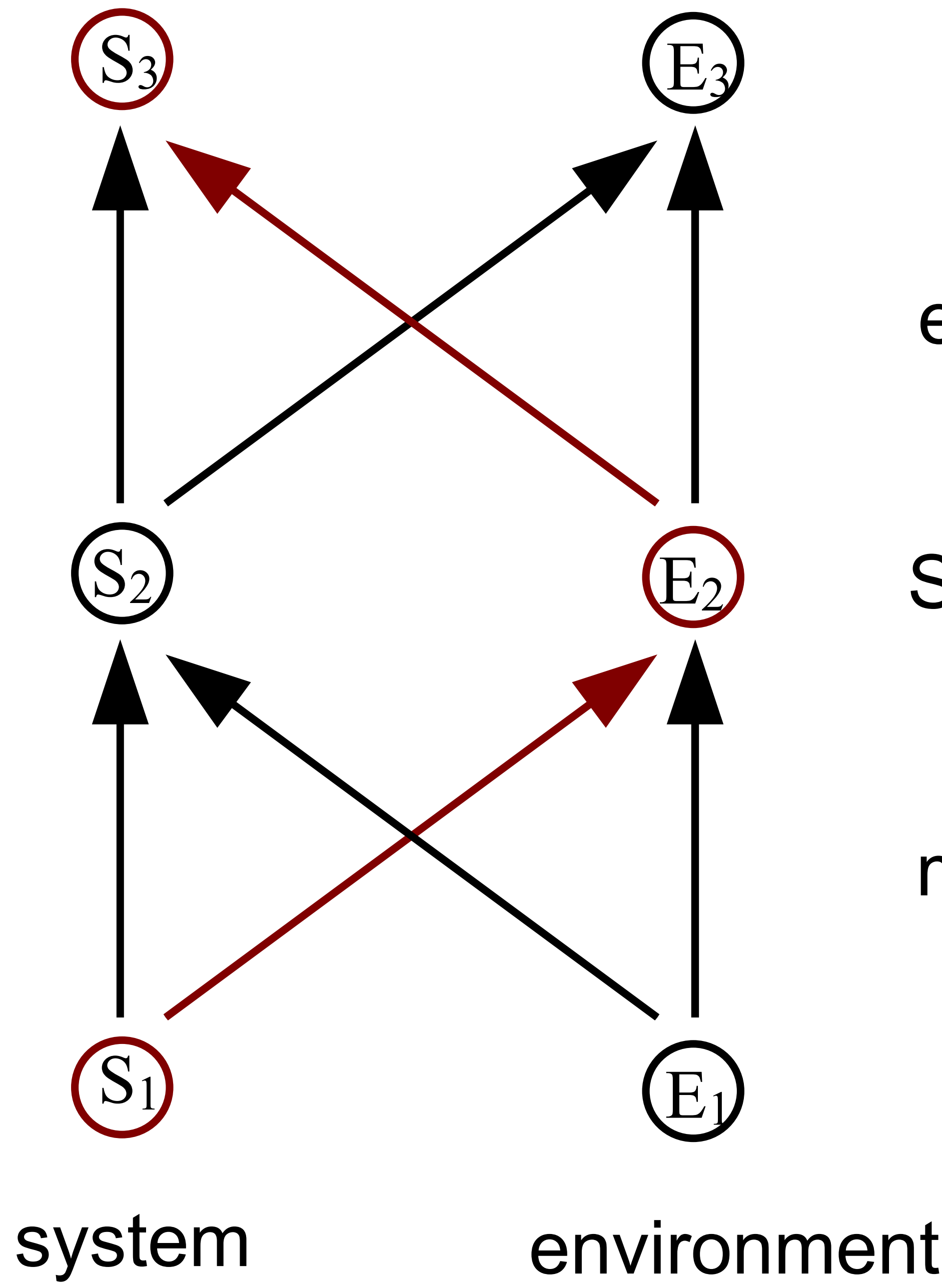
# 6. Application

how causal inference relates to open quantum system dynamics

# Evolution of an open (quantum) system



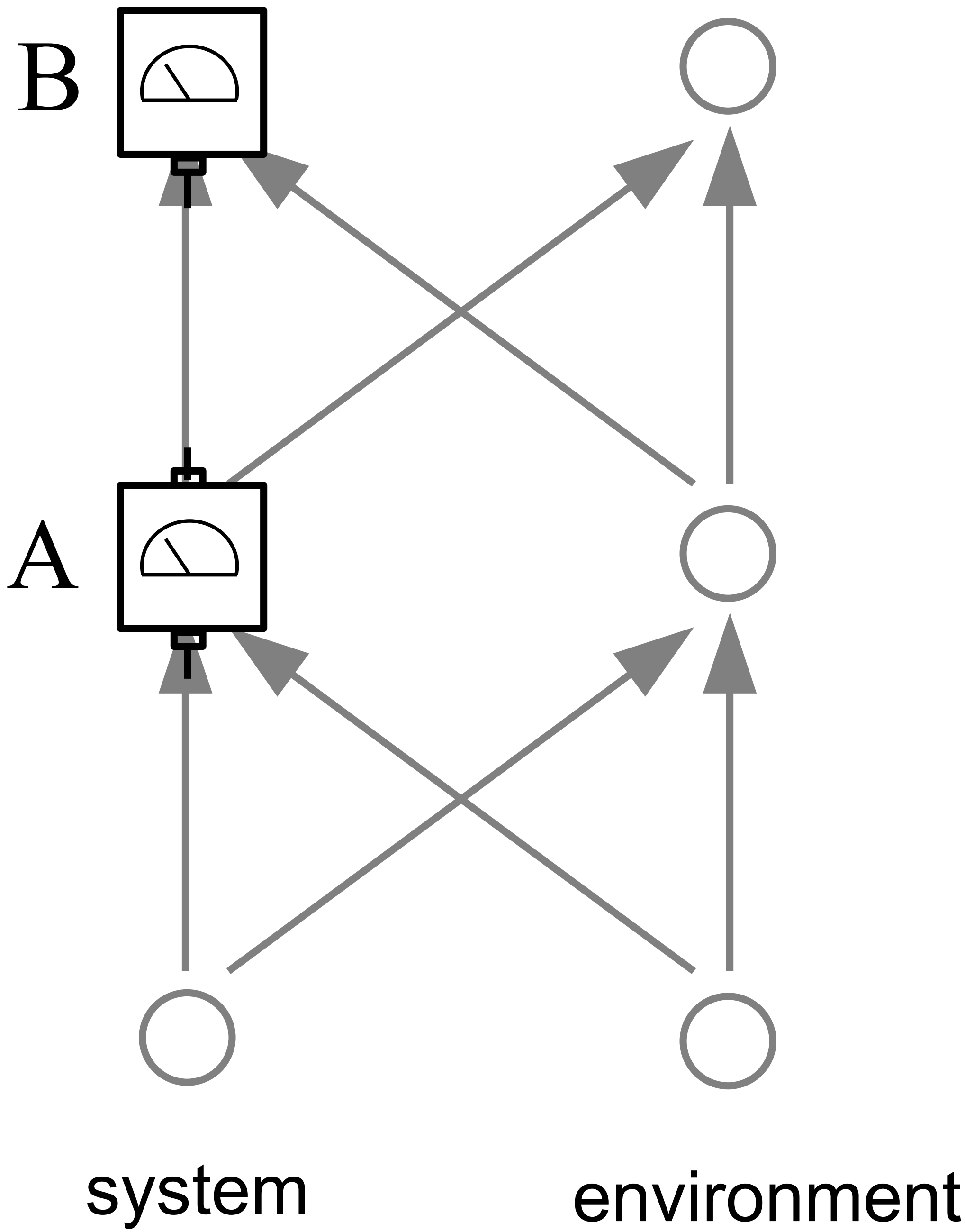
# Evolution of an open (quantum) system

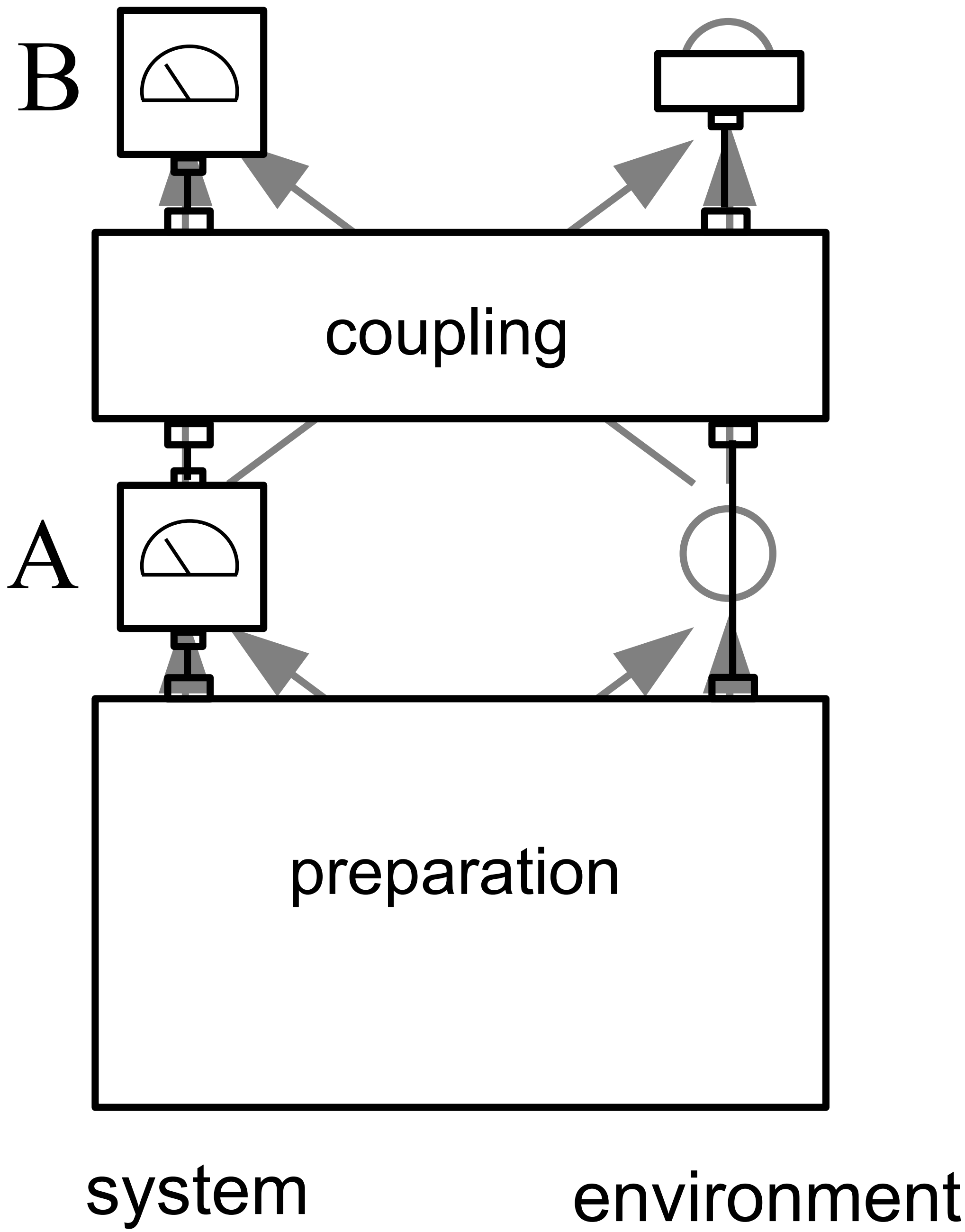


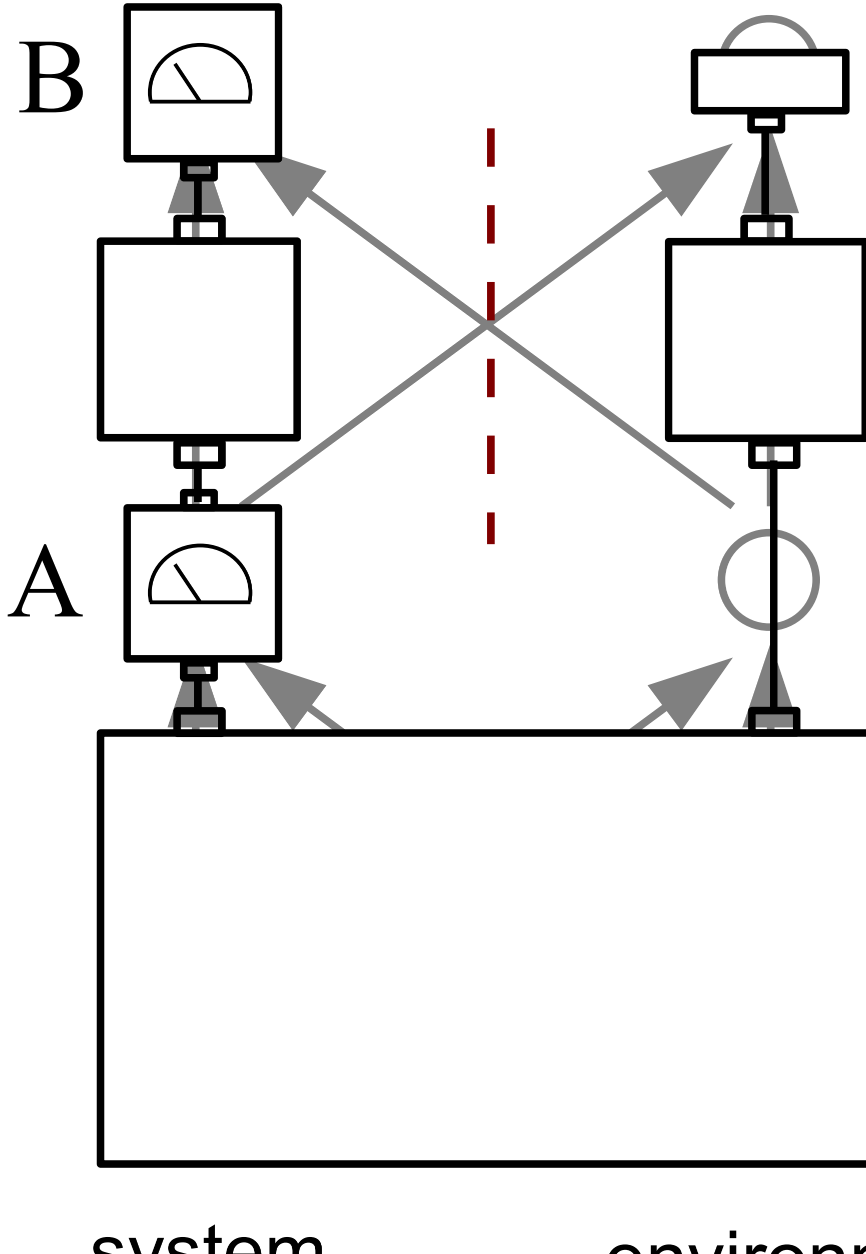

environmental back-action

$S_3$ depends on $S_2$ and $S_1$

memory effect


B
A
system
environment


B
coupling
A
preparation
system
environment

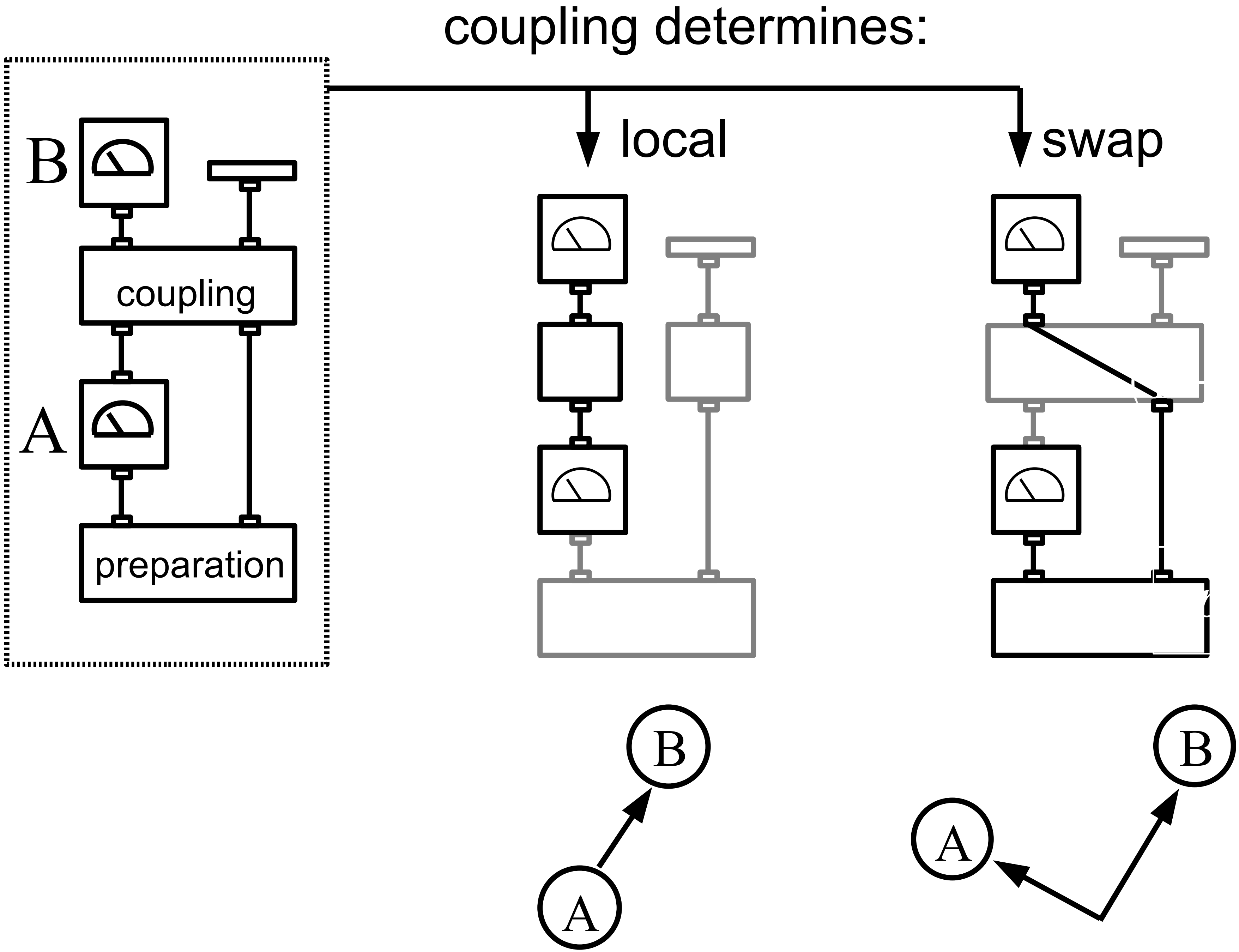



purely cause-effect relation between A and B

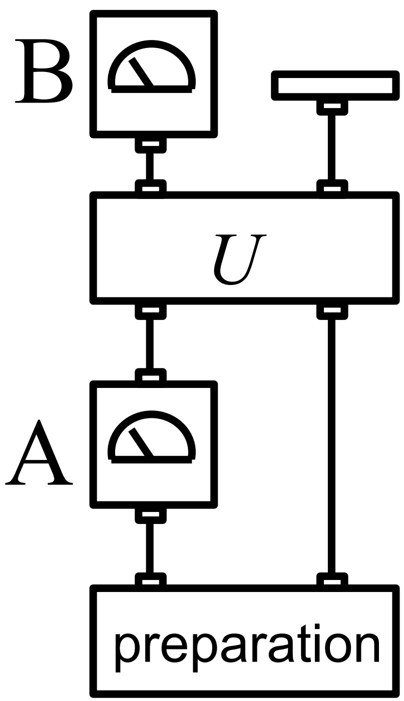
⇒no back-action from environment

# 8. Outlook

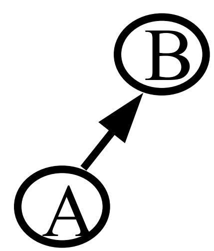
superpositions of causal structures



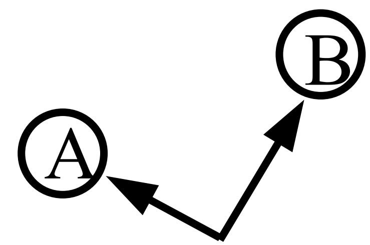
coupling determines:
local
swap
B
A
coupling
preparation
B
A
B
A




$$U = \cos\theta\,\mathbf{1} + i\,\sin\theta\,S$$




*and*




"coherent" (?)

# Highlights

- program: reconcile classical notion of causality with QT
  - provides new perspective on 'quantumness'
- the quantum advantage:
  - classically, information symmetry prevents causal inference
  - quantum correlations can reveal causal structure
  - quantum advantage for novel kind of task
- tabletop experiment with tunable causal structure
- application as test of Markovianity
- circuit that 'superposes' two causal relations

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