A contextual Wigner model for qubit stabilizer QM

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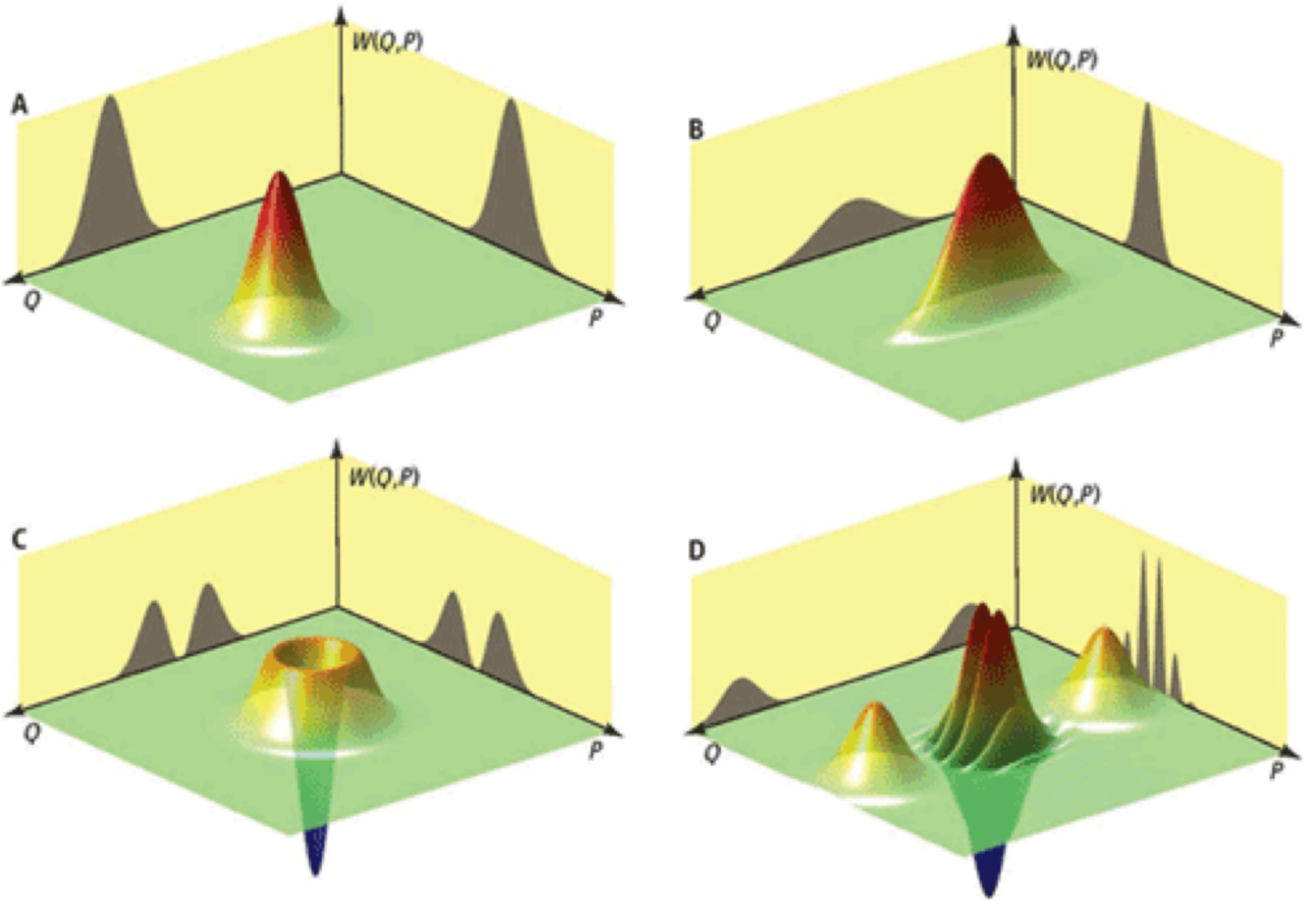
A contextual model based on:

Wigner functions



A
W(Q,P)
Q
P
B
W(Q,P)
Q
P
C
W(Q,P)
Q
P
D
W(Q,P)
Q
P

Wigner function

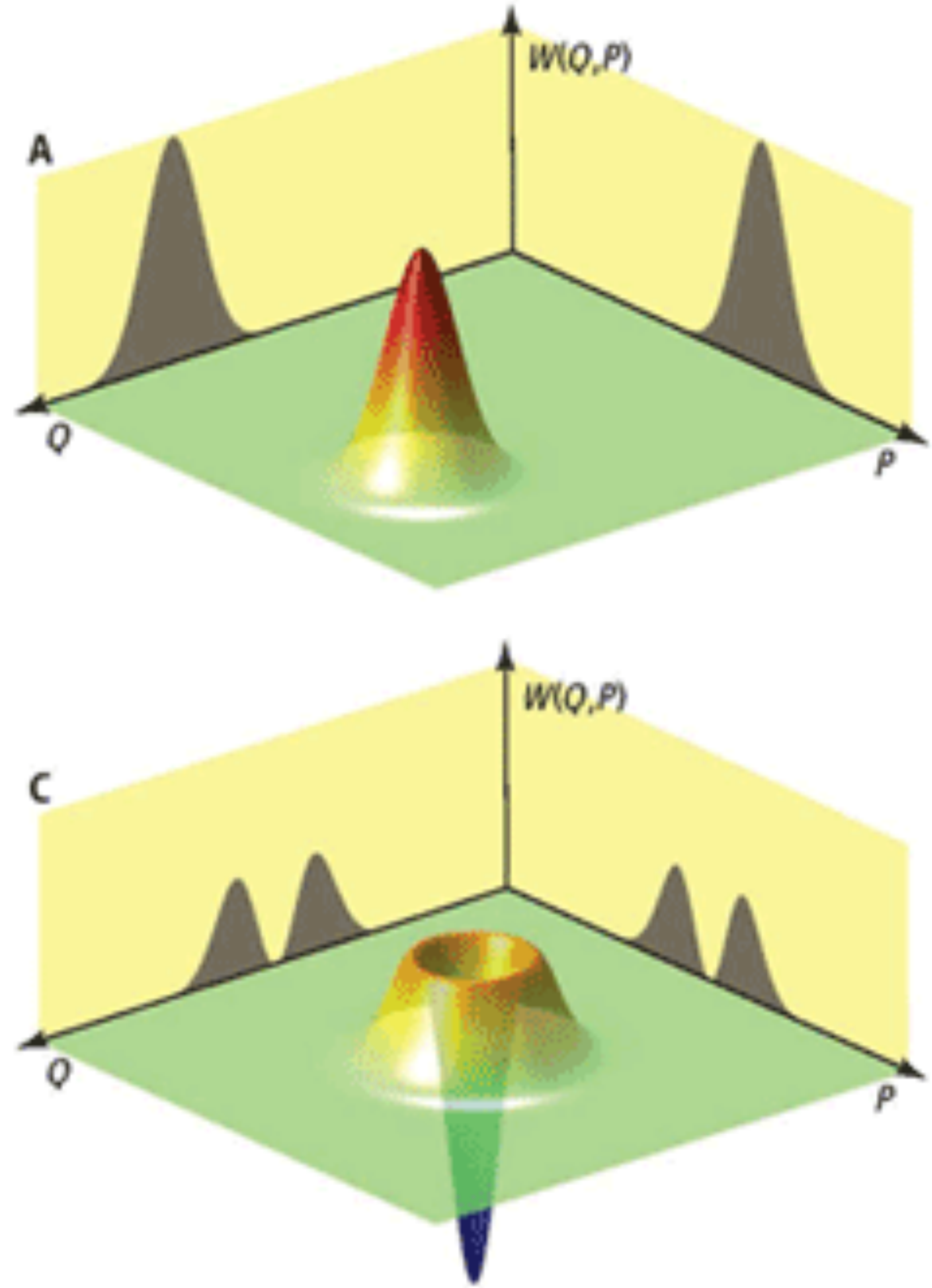
- Wigner (1932)
- A real valued representation of a state in **phase space**
    - *e.g. position / momentum*

Density operator

↕

Wigner function

- Not unique, many different "Wigner functions"

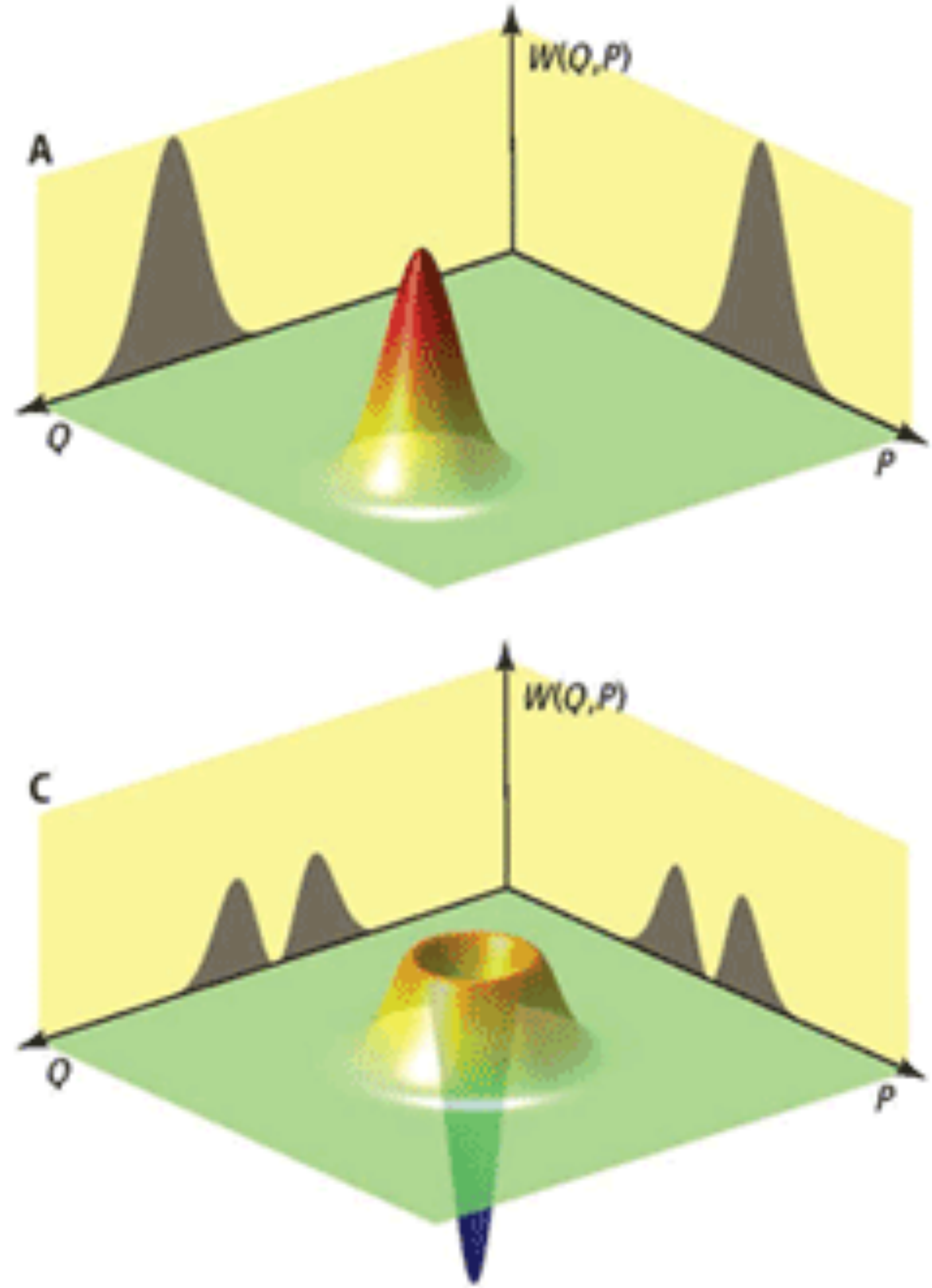


Wigner function

- A **quasi-probability distribution**
- May take **negative values**
- **Integrating out** one variable leaves a **probability distribution.**

pure state + non-negative Wigner function
<-> pure Gaussian state

— *Hudson's theorem*



Wootters (1987): Wigner functions for d-dimensional systems (**qudits**).

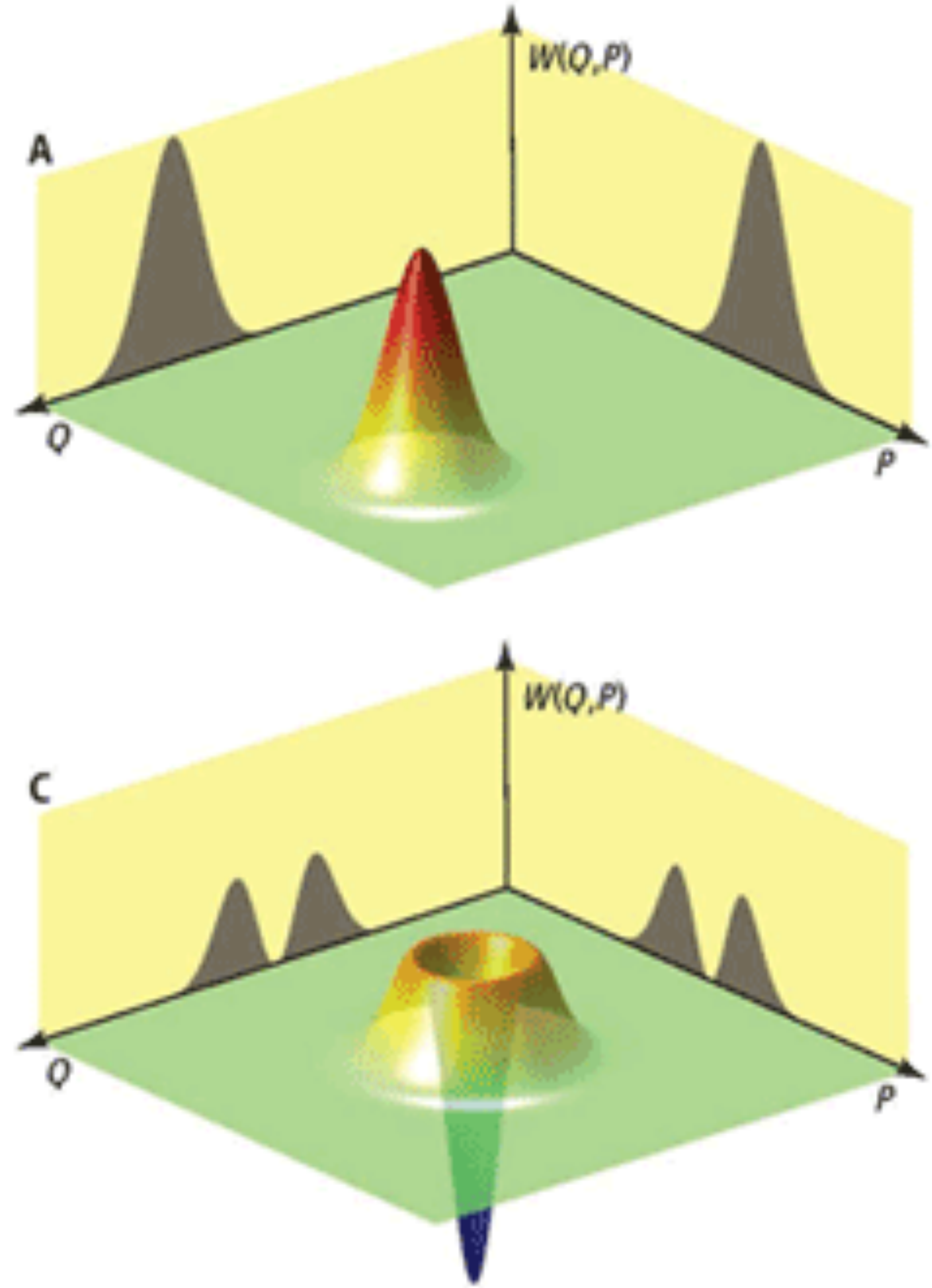
- Phase space:
  - d x d real valued `grid'
- E.g. for a qutrit (d=3)

0.3	0.7	-0.2
-0.2	0	-0.3
0.4	-0.3	0.7

"**Negativity** and **contextuality** are equivalent notions of nonclassicality"

— **Spekkens** (2008):

- A quantum sub-theory describable with:
  - solely *non-negative* Wigner functions
  - has a *non-contextual hidden variable model* (NCHVM).



Gross (2006): Finite dim. Hudson's theorem

For qudits of **odd dimension** d:

pure state + non-negative Wigner function

<-> Pure stabilizer state

Gross (2006): Finite dim. Hudson's theorem

For qudits of **odd dimension** d:

pure state + non-negative Wigner function

<-> Pure stabilizer state

- (via Spekkens) Implies that, in odd d, stabilizer QM is *non-contextual.*

Stabilizer quantum mechanics

Stabilizer States:

Common eigenstates of **commuting Pauli operators**

E.g. The Bell states, e.g.:

$$X \otimes X|\psi\rangle = |\psi\rangle$$
$$Z \otimes Z|\psi\rangle = |\psi\rangle$$

$$|\psi\rangle = |00\rangle + |11\rangle$$

Pauli operators can be generalised to d-dim. qudit systems:

$$X = \sum_j |j \oplus 1\rangle\langle j|$$

$$Z = \sum_j \omega^j |j\rangle\langle j|$$

where $\omega = e^{i2\pi/d}$

and thus we can define stabilizer QM for qudits.

Stabilizer quantum mechanics

Measurements:

Pauli observables

Unitary transformations:

Clifford group

→ Set of unitaries that map Paulis to Paulis

Generated by:

* Hadamard, CNOT gate, S gate

(and qudit analogues)

Stabilizer quantum mechanics

n qubit state:
n **stabilizer generators** = full state description.

Stabilizer quantum mechanics can be **efficiently classically simulated.**

— Gottesman-Knill theorem

Stabilizer computation can be boosted to **quantum universality** with "magic state" resources.

2 is the oddest prime

d = odd

- Stabilizer QM is **non-contextual**
- Can be fully represented in **non-negative** Wigner functions.

d = 2

- Qubit stabilizer QM exhibits **contextuality**
- Even **state-independent** contextuality - e.g. Mermin's square.

Mermin's square

Consider the following **square** of observables:

$$\begin{bmatrix} X \otimes I & I \otimes X & X \otimes X \\ I \otimes Z & Z \otimes I & Z \otimes Z \\ X \otimes Z & Z \otimes X & Y \otimes Y \end{bmatrix}$$

- Each row and column: **3 commuting operators.**
- Measuring operators from a row or a column, will prepare a joint eigenstate, *regardless of initial state.*

Mermin's square

A non-contextual assignment is impossible:

$$\begin{bmatrix} X \otimes I & I \otimes X & X \otimes X \\ I \otimes Z & Z \otimes I & Z \otimes Z \\ X \otimes Z & Z \otimes X & Y \otimes Y \end{bmatrix}$$

Key point:

$$(X \otimes X)(Z \otimes Z) = -(Y \otimes Y)$$

but

$$(X \otimes Z)(Z \otimes X) = (Y \otimes Y)$$

Qubit stabilizer QM is contextual.

-> No non-negative Wigner model for all of qubit stabilizer QM.

Qubit Wigner functions

Nevertheless, we can still define qubit Wigner functions (Wootters (1987)):

$$W(u) = \frac{1}{2}\,\mathrm{Tr}[\rho A(u)] \qquad\qquad \rho = \frac{1}{4}\sum_u A(u)W(u)$$

``Phase point'' operators:

$$\begin{aligned}
A(00) &= I + X + Y + Z \\
A(01) &= I + X - Y - Z \\
A(10) &= I - X - Y + Z \\
A(11) &= I - X + Y - Z
\end{aligned}$$

Qubit Wigner functions

Examples: (As a square or column as convenient...)

$$W_\rho : \begin{bmatrix} W(00) & W(01) \\ W(10) & W(11) \end{bmatrix} \text{ or } \begin{bmatrix} W(00) \\ W(01) \\ W(10) \\ W(11) \end{bmatrix}$$

Qubit Wigner functions

One qubit stabilizer states:

$$W_{|0\rangle\langle 0|} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \qquad W_{|1\rangle\langle 1|} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$W_{|+i\rangle\langle +i|} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

All 6 one-qubit stabilizer states have a non-negative representation.

Qubit Wigner functions

But:

$$\rho = \frac{1}{2}\left(I + \frac{1}{\sqrt{3}}(X + Y + Z)\right)$$

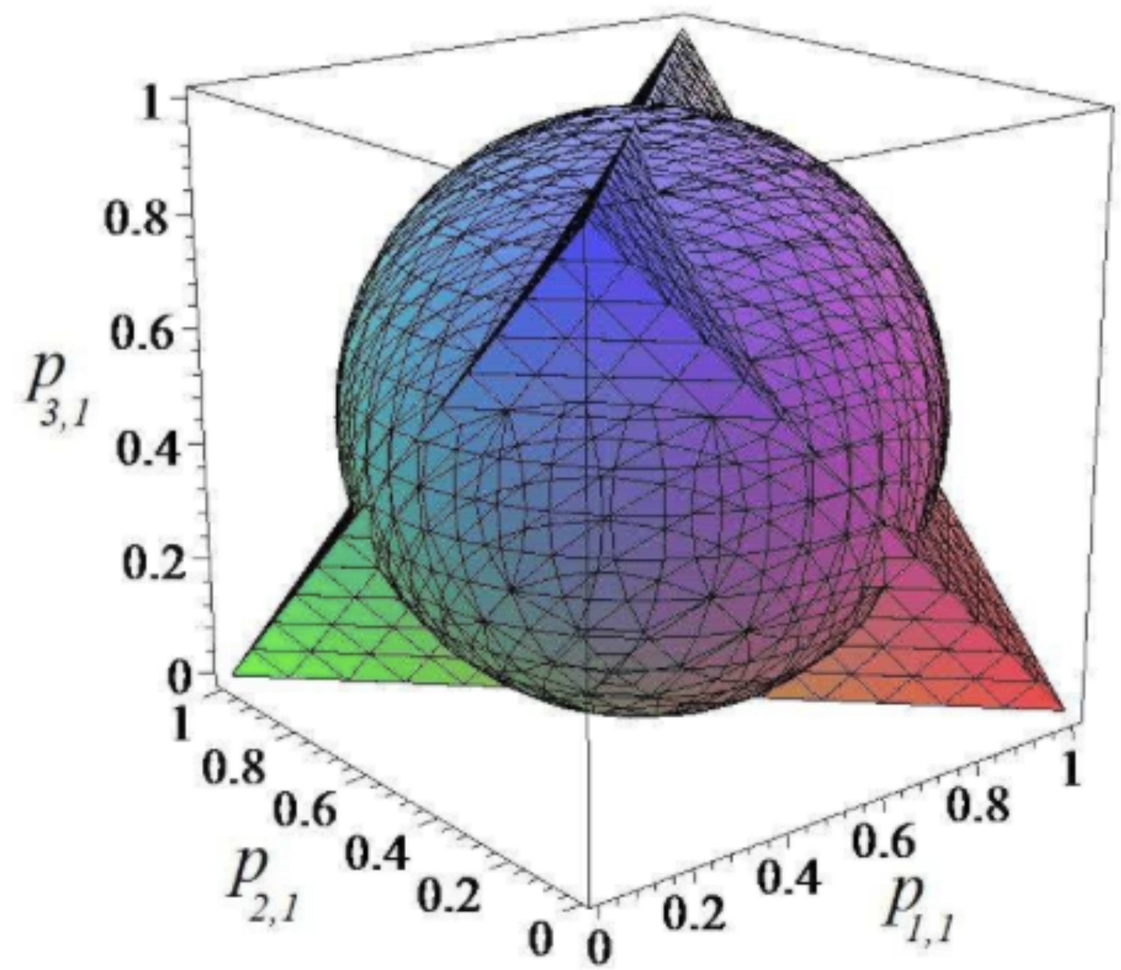
A "magic state" maximally far from the stabilizer states is **also** non-negative.

$$W_\rho = \begin{bmatrix} 1.36 & 0.21 \\ 0.21 & 0.21 \end{bmatrix}$$

Qubit Wigner functions

Galvao (2005):

- Tetrahedron: The set of non-negative "density matrices"
- Sphere: Set of physical density matrices



Qubit Wigner functions

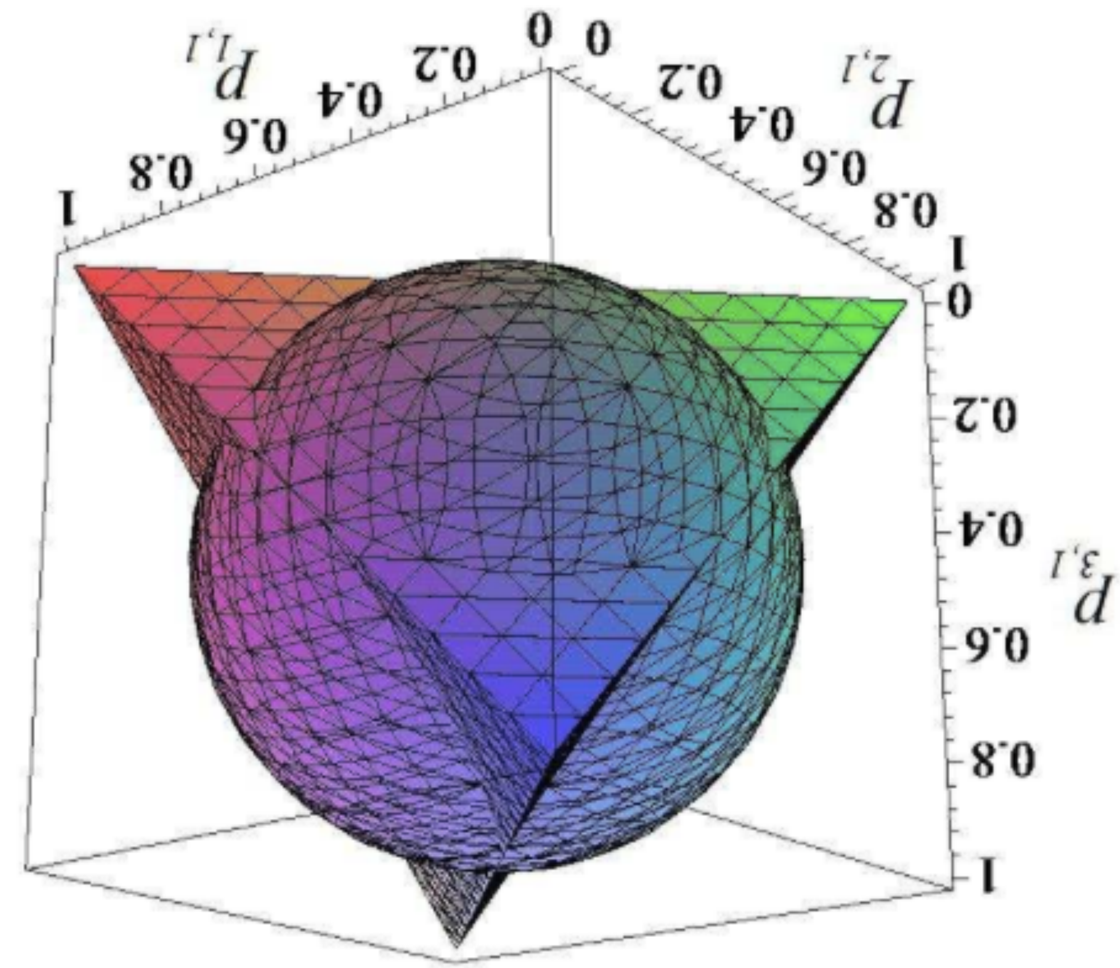
But Wigner function is not unique. Consider two Wigner functions:

$$\underbrace{\begin{aligned} A(00) &= I + X + Y + Z \\ A(01) &= I + X - Y - Z \\ A(10) &= I - X - Y + Z \\ A(11) &= I - X + Y - Z \end{aligned}}_{W^+} \quad \Bigg| \quad \underbrace{\begin{aligned} A(00) &= I + X - Y + Z \\ A(01) &= I + X + Y - Z \\ A(10) &= I - X + Y + Z \\ A(11) &= I - X - Y - Z \end{aligned}}_{W^-}$$

Qubit Wigner functions

Galvao (2005):

- Tetrahedron: The set of non-negative "density matrices" for W^-
- Sphere: Set of physical density matrices



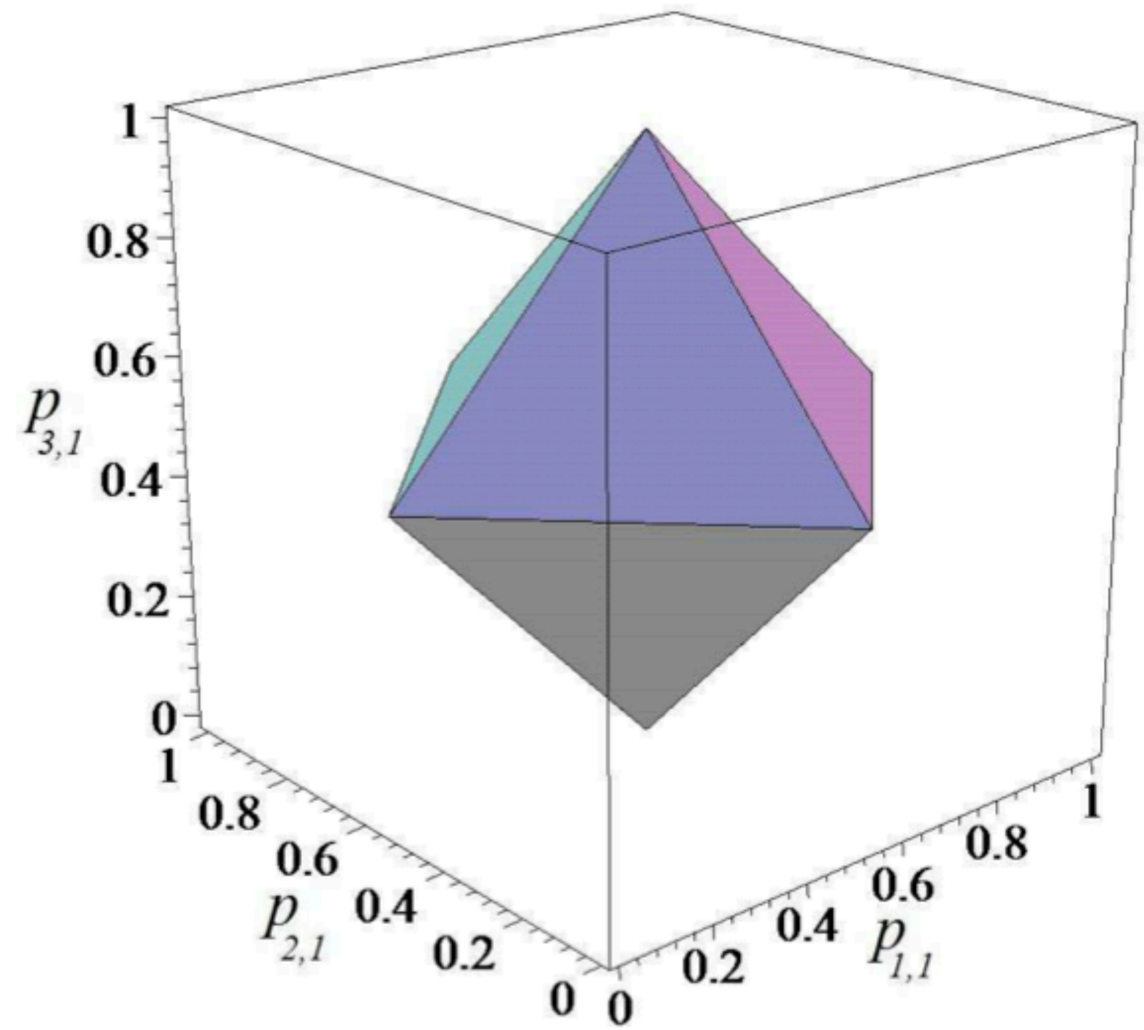
Qubit Wigner functions

Galvao(2005)

- For single qubit states, precisely the stabilizer states are **non-negative** for **both** Wigner functions.

Conjecture: Wigner negativity a necessary resource for quantum computational speed-up

->Howard, Wallman, Veitch, Emerson, (2014) *"Contextuality provides the "magic".*



Qubit Wigner functions

- For multi-qubit states it gets more complicated, but many general results can be derived. Clifford group still plays a prominent role. (Cormick, Galvao et al, (2005))

- If we restrict to **rebits** and a sub-group of the Clifford group, we recover a positive theory. *Delfosse et al (2015)*

Qubit Wigner functions

- Whichever Wigner function is chosen:
  - contextuality -> **some** stabilizer states must have a **negative** representation

This talk:

A contextual non-negative Wigner theory for qubit stabilizer QM.


Work in progress...

Why would we want one?

- Provide a **contextual hidden variable model** for qubit stabilizer QM.
- Help us understand:
  - "stabilizer-like" models such as **Spekkens' toy model**
  - why / how $d = 2$ and $d =$odd **differ** so much, and **unify.**
  - the role of contextuality in *qubit* **magic state** QC.
- Wigner function a powerful tool.
  E.g. Generalise **Gottesman-Knill**?

Qubit Wigner functions in more detail

1. For our n qubit state space, we define a **Hermitian** basis of **Pauli operators:**
2. Each operator associated with a point in **phase-space** $u = u_z u_x$.

$$T(u) = \underbrace{i^{u_x \cdot u_z \pmod 2}}_{\text{arbitrary convention}} Z(u_z)X(u_x)$$

Notation: $Z(u) = \bigotimes_j Z^{u_j}$. E.g. $Z(101) = Z \otimes I \otimes Z$, $X(10) = X \otimes I$.

Qubit Wigner functions revisited

E.g. for a single qubit:

$$\begin{bmatrix} T(00) & T(01) \\ T(10) & T(11) \end{bmatrix} = \begin{bmatrix} I & X \\ Z & -Y \end{bmatrix}$$

E.g. for 2 qubits:

$$T(1000) = Z \otimes I$$
$$T(1111) = -Y \otimes Y$$

We make a **particular choice** of basis operators:

$$A(u) = \sum_u \pm T(u)$$

Inspired by David Gross "universal" qudit Wigner function.

Wigner function : expectation values wrt this orthogonal set of Hermitian operators.

$$W(u) = \frac{1}{2^n} \mathrm{Tr}[\rho A(u)] \qquad \rho = \frac{1}{4^n} \sum_u A(u) W(u)$$

The sign choices can be captured in a Boolean function $F(u)$,

$$A(0) = \sum_u (-1)^{F(u)} T(u)$$

One can show that the following set

$$A(u) = T(u) A(0) T(u)$$

for all phase points u, is pairwise orthogonal:

$$\mathrm{Tr}[A(u) A(v)] = 8^n \delta_{u,v}$$

Many Boolean functions $F(u)$.

Many Wigner functions.

Which do we choose?

Wigner frame

All of them!

Consider each as a different "frame of reference".

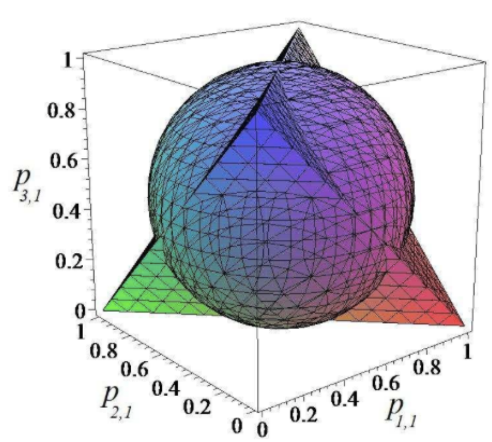
Call Boolean function $F(u)$ the **Wigner frame:**

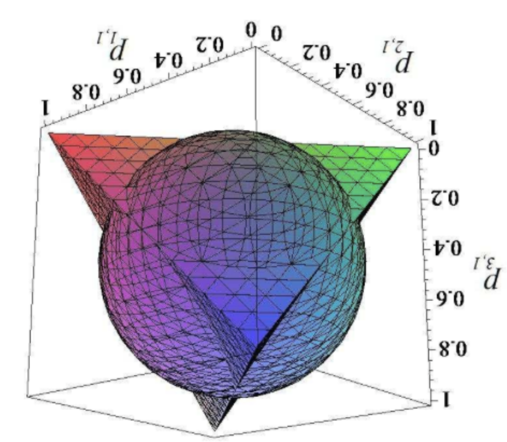
- Add the frame to the state description:
- Wigner function is now defined within a given **frame.**

Wigner frame

For n qubits, there are $2^{2^{2n}-2n-1}$ Wigner frames (up to $T(u)$ symmetries).

Example:
For $n = 1$ there are 2 frames W^+ and W^-.





Key idea

- Each Wigner **frame** has its own set of **non-negative** stabilizer states.
- Construct a non-negative Wigner theory for **all** stabilizer states by *changing frame* to preserve positivity.
- The need to **change frame** embodies the **contextuality.**

Aim

Construct an accurate and complete model of qubit stabilizer QM with **non-negative Wigner functions**, including:

1. state description (non-negative Wigner function + frame)
2. unitary state update rules
3. measurement rules: probability and state update

States

States: Single qubit

The W^+ frame: $F(x) = u_1^z u_1^x$

$$W_{|0\rangle\langle 0|} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \quad W_{|1\rangle\langle 1|} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$W_{|+\rangle\langle +|} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad W_{|-\rangle\langle -|} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$W_{|+i\rangle\langle +i|} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad W_{|-i\rangle\langle -i|} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

States: Single qubit

The W^- frame: $F(x) = 0$

$$W_{|0\rangle\langle 0|} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} W_{|1\rangle\langle 1|} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$W_{|+\rangle\langle +|} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} W_{|-\rangle\langle -|} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$W_{|+i\rangle\langle +i|} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} W_{|-i\rangle\langle -i|} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

States: Two qubit

Represent Wigner functions as a square:

$$\begin{bmatrix} W(0000) & W(0001) & W(0100) & W(0101) \\ W(0010) & W(0011) & W(0110) & W(0111) \\ W(1000) & W(1001) & W(1100) & W(1101) \\ W(1010) & W(1011) & W(1110) & W(1111) \end{bmatrix}$$

Examples:

Frame: $F(u) = 0$

- **State:** $|0\rangle|0\rangle$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Examples:

Frame: $F(u) = 0$

→ **Bell state:** $|0\rangle|0\rangle + |1\rangle|1\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Examples:

Frame: $F(u) = 0$

- Cluster state: $|0\rangle|+\rangle + |-\rangle|1\rangle$

$$\begin{bmatrix} 0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$$

Negative!

Examples:

But in frame: $F(u) = u_1^z u_2^z u_1^x u_2^x$

- **Cluster state:** $|0\rangle|+\rangle + |-\rangle|1\rangle$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Non-Negative!

Each frame has a set of *non-negative* stabilizer states.

We can show that:

* **every** stabilizer state is positive in at least one Wigner frame.

We can write down:

* expressions for **all** frames that are positive for a given state.

Gates

Gates

Every Clifford unitary transformation can be reduced to:

* a permutation of the Wigner function
* a change of Wigner frame [4]

To ensure that:

* non-negative state mapped to non-negative state.

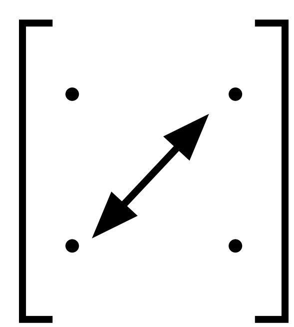
[4] These may be frame dependent.

E.g. Hadamard

$$|0\rangle \leftrightarrow |+\rangle \qquad |1\rangle \leftrightarrow |-\rangle \qquad |+i\rangle \leftrightarrow |-i\rangle$$

Implementation in our Wigner model:

- Permutation of Wigner function:



- With frame change: $W^+ \leftrightarrow W^-$

Gates

- Can derive similar rules for the other generators of the Clifford group.

Measurement

Measurement

- Let ρ be a stabilizer state
- Let Π be an effect for a Pauli observable σ.

$$\Pi = (I \pm \sigma)/2$$

Measurement

Two kinds of measurement in stabilizer QM.

1. $[\rho, \Pi] = 0$: Measurement `learns'
   - $\rho = I$, $\sigma = Z$: $\rho \to |0\rangle\langle 0|$
2. $[\rho, \Pi] \neq 0$: Measurement `disturbs' [3]
   - $\rho = |0\rangle\langle 0|$, $\sigma = X$: $\rho \to |+\rangle\langle +|$

---

[3] If there are any classics scholars here, I'd appreciate suggestions for terminology!

Measurement

We exploit the **duality** between states and measurements.

$$\Pi = (I \pm \sigma)/2$$

- Effect Π is a **projector** and has its **own** Wigner function.

Wigner function Born rule

- Let $W(u)$ be the Wigner function for ρ.
- Let $\Pi(v)$ be the Wigner function for Π.

$$\mathrm{Tr}[\rho\Pi] = \frac{1}{2^n}\sum_u W(u)\Pi(u)$$

Wootters (1987)

Measurement update rule

$$[\rho, \Pi] = 0$$

It the commuting case, we can derive a simple measurement update rule:

Measurement update rule $[\rho, \Pi] = 0$

Choose a frame $F(x)$ where:

- **state** ρ
- **effect** Π
  - **and resultant state** $\rho' \propto \Pi\rho\Pi$

are all non-negative. Then:

$$W_{\rho'}(u) = W_{\rho}(u) W_{\Pi}(u)$$

Mermin square

$$\begin{bmatrix} XI & IX & XX \\ IZ & ZI & ZZ \\ XZ & ZX & YY \end{bmatrix}$$

Each row and column defines an experiment:

- Three commuting Pauli measurements

Or equivalently:

- prep. of a pure stabilizer state + a Pauli measurement

Mermin square

$$\begin{bmatrix} XI & IX & XX \\ IZ & ZI & ZZ \\ XZ & ZX & YY \end{bmatrix}$$

Hence we need to describe:

- 6 (x 4) pure stabilizer states
- 5 Pauli effects - *(can prove these are non-neg in all frames)*

6 pure stabilizer states: - Frame: $F(u) = 0$

$$XI, IZ : \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} IX, ZI : \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} XX, ZZ : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$XI, IX : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ZI, IZ : \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$ZX, XZ : \begin{bmatrix} 0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$$

6 pure stabilizer states: - Frame: $F(u) = u_1^z u_2^z u_1^x u_2^x$

$$XI, IZ : \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} IX, ZI : \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$XX, ZZ : \begin{bmatrix} 0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$XI, IX : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ZI, IZ : \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ZX, XZ : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In frame: $F(u) = 0$

$$XX, ZZ : \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$(1 + YY)/2$ **effect:**

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

In frame: $F(u) = u_1^z u_2^z u_1^x u_2^x$

$$ZX, XZ: \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$(1 + YY)/2$ **effect:**

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Measurement when $[\rho, \Pi] \neq 0$

To summarise

We have derived a model for qubit stabilizer QM with:

- state description
- unitary state update rules
- measurement probability and update rules

where Wigner description is **non-negative** at all stages.

A contextual hidden variable model: **Frame** choice and dependence embodies the **contextuality**.

Something about this is familiar...

Spekkens' Toy model

- A *non-contextual* HV model for qubit-like systems[5]
- Derived from a "**knowledge balance principle**"

"If one has maximal knowledge, then for every system, at every time, the amount of knowledge one possesses about the ontic state of the system at that time must equal the amount of knowledge one lacks."[5]

- Reproduces **many** features of stabilizer QM:
  - but is not QM, some predictions **contradict** QM.

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[5] Rob Spekkens, Phys. Rev. A 75, 032110 (2007)

Spekkens' Toy model

- Can be described in the same category theoretic frame-work as stabilizer QM (QPL '09).

Phase groups and the origin of non-locality for qubits

Bob Coecke, Bill Edwards and Robert W. Spekkens

Oxford University Computing Laboratory

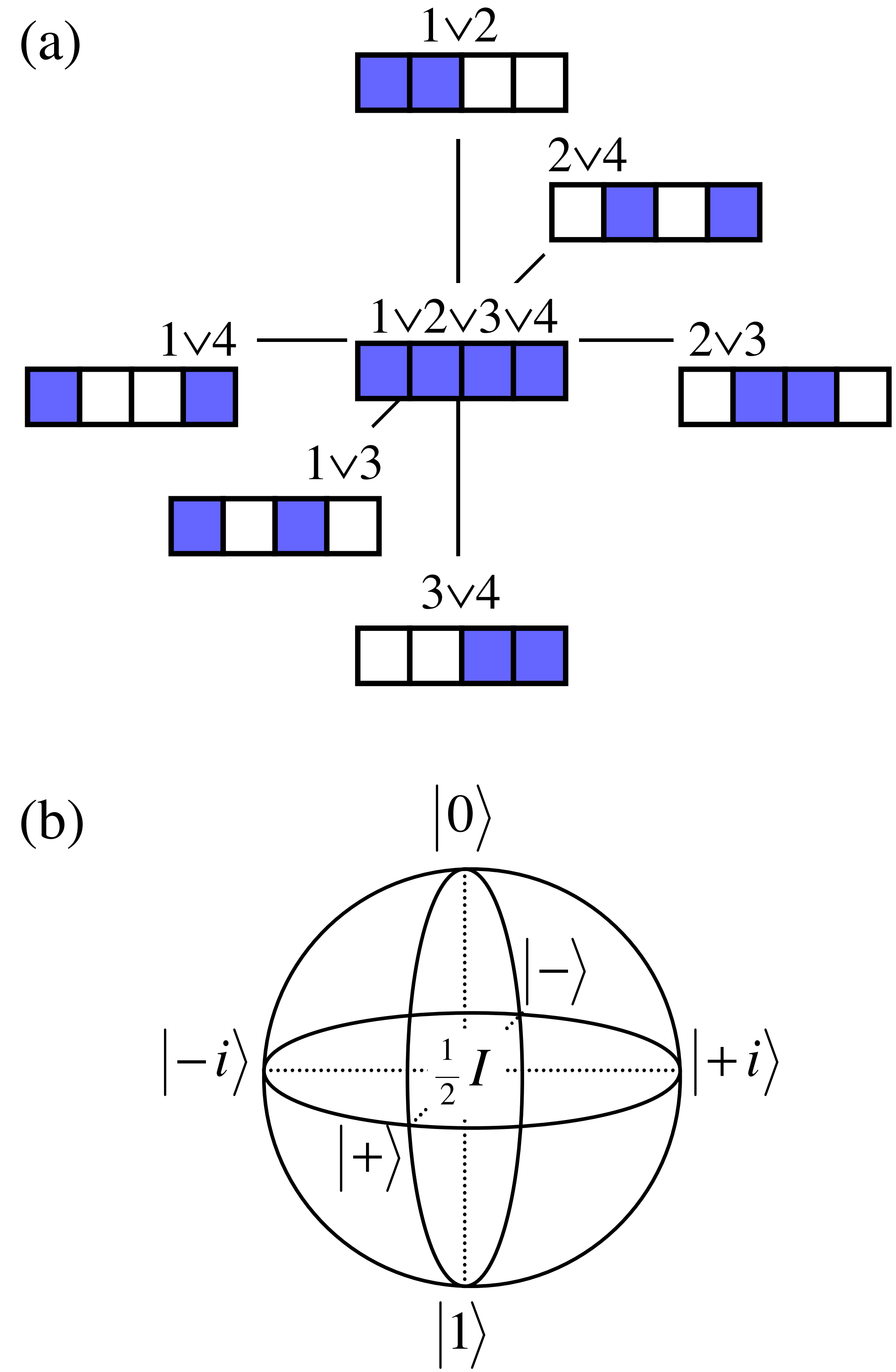
States

Epistemic states and ontic states

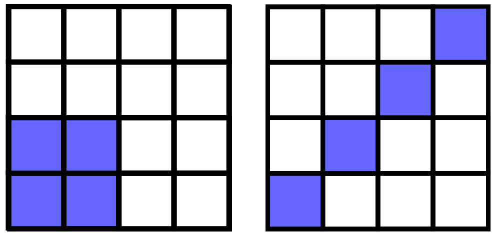
Knowledge-balance:

- We may know at most *half* the information about ontic state.

E.g. for the qubit-like system **(Spekkens-bit)**, 4 ontic states (hidden variables), 6 epistemic states:



Two-Spekkens-bit states have the form:

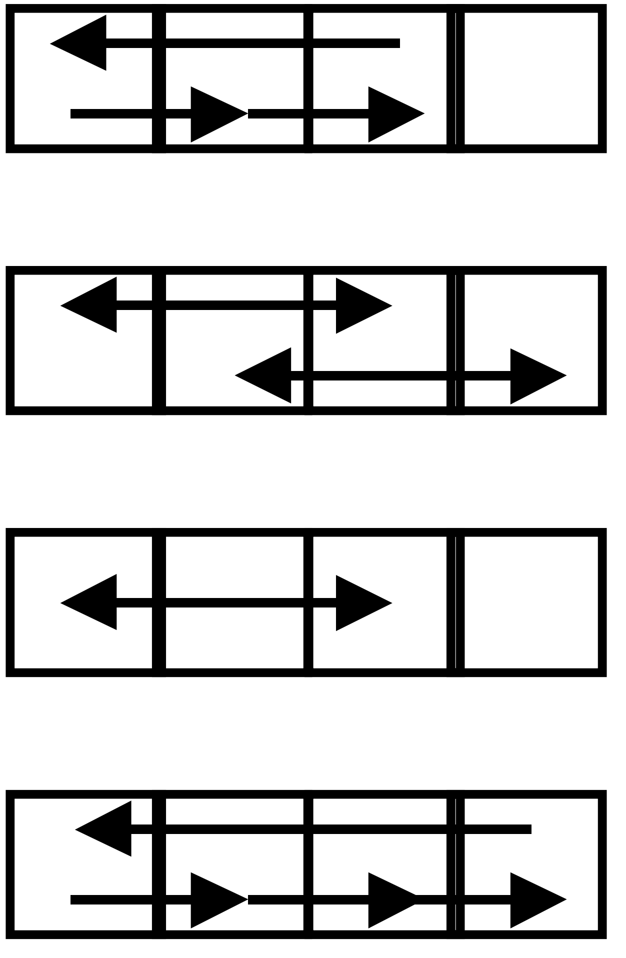


Knowledge balance principle allows one to derive allowed states for any number of subsystems.

Gates

Transformations

Permutations of the ontic states:



Measurements

E. Measurement update rule

Suppose the initial epistemic state is $1 \vee 2$, a reproducible measurement of $1 \vee 3$ versus $2 \vee 4$ is performed, and the outcome $1 \vee 3$ occurs. In this case, one can retrodict that the ontic state of the system must have been 1 prior to the measurement. This is not in conflict with the knowledge balance principle since the latter does not place restrictions on what one can know, at a given time, about the ontic state at an earlier time. The principle *does*, however, place restrictions on what one can know, at a given time, about the ontic state at that time. If it were the case that the system's ontic state was known to be unaltered in the process of measurement, then one's description of the system prior to the measurement would apply also after the measurement. But then, one would know the system to be in the ontic state 1 after the measurement, and this *is* in violation of the knowledge balance principle. Since we assume that information gain through measurements is always possible, we must conclude that measurement causes an unknown disturbance to the ontic state of the system.

In our particular example, the assumption that the the system) or the permutation (ab)(c)(d) occurs (if the ontic state is a, it becomes b and vice-versa), but it is unknown which.

Note that the possible permutations resulting from a measurement depend only on the identity and outcome of the measurement and not on the initial epistemic state. This is appropriate, since the nature of someone's knowledge of a system should not influence how the ontic state of the system evolves during a measurement. By the same token, whether or not the system is initially correlated with other systems should not influence the nature of the evolution of the ontic state of the system during a measurement, because the presence or absence of such correlation is a feature of an observer's knowledge of the system, not a property of the system itself [2]. Thus, although we have derived the nature of the unknown disturbance by considering an example where the system being measured is not correlated with any other system, the results obtained must also be applicable when such correlation is present. We will therefore make use of the results derived above when we consider measurements on one member of a pair of systems in Sections IV A, IV B and IV G.

Measurements

Measurement rules derived from the **knowledge balance principle.**

Yes - it did look familiar...

- Transform our **contextual non-negative Wigner theory** into a **non-contextual** one:
  - **Fix the frame** and extend the set of non-negative Wigner states to all Wigner functions of correct form.
  - **Remove frame dependence** in measurement and transformation rules.

i.e. "Turn off" the contextuality

- If we "turn contextuality off", and compare with Spekkens:

States: Epistemic states allowed by *k-b-p* **coincide** with valid non-negative Wigner functions. ✅

Transformations: In both cases, **permutations** of "ontic states" / phase points. ✅

Measurements:

- Commuting case: Update rule and probability rules **coincide** ✅
- Non-commuting case: Work-in-progress. Analysis so far suggests they **coincide.** 😌

Deriving Spekkens' model from QM

- Start with: **Contextual non-negative Wigner model** of qubit stabilizer QM.
- **"Turn off"** the contextuality
  - (ignore frames and frame dependence)
- Recover (*it seems*) **Spekkens' model.**

Summary

- We developed a simple **non-negative Wigner function model** for qubit stabilizer QM.
- **Contextuality** embodied by "**Wigner frames**".
- By "**switching off** the contextuality" we (seem to) recover precisely **Spekkens' toy model.**

Outlook

- Finish off the **technical proofs** (esp. non-commuting meas.)
- Describe model **without** quantum mechanical machinery.
- **Complexity** of this model for simulation?
- What about the **non-negative non-stabilizer states**?
  Extend **Gottesman-Knill**?
- Use model to analyse to **magic state** QC.

Outlook

- Link with other **frameworks** for contextuality e.g. Sheaf-Theoretic approach of Abramsky et al.
- Spekken's toy model is axiomatized. **Axiomatize** stabilizer QM?
- Distinguish "**stabilizer** contextuality" from "**general** contextuality"?
- Where does **STAB vs SPEK** fit in?
- ...

Thank you!

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