Verifying Compilers using Multi-Language Semantics

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Semantics-preserving compilation

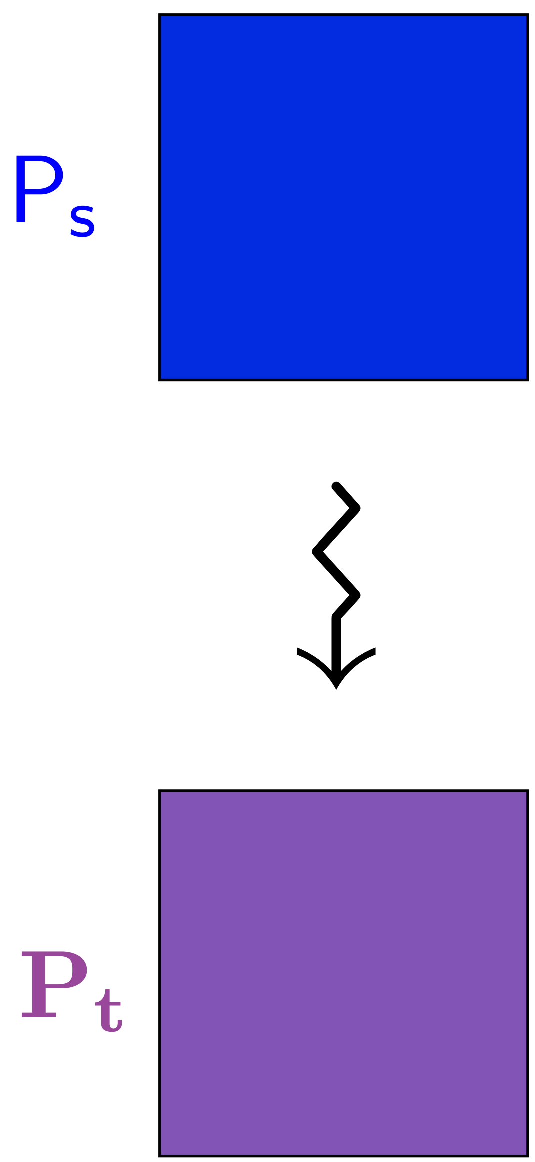
$$s \leadsto t \quad\Longrightarrow\quad s \approx t$$

compiles to ($\leadsto$)

same meaning ($\approx$)

Problem: Closed-World Assumption

Correct compilation guarantee only applies to **whole** programs!



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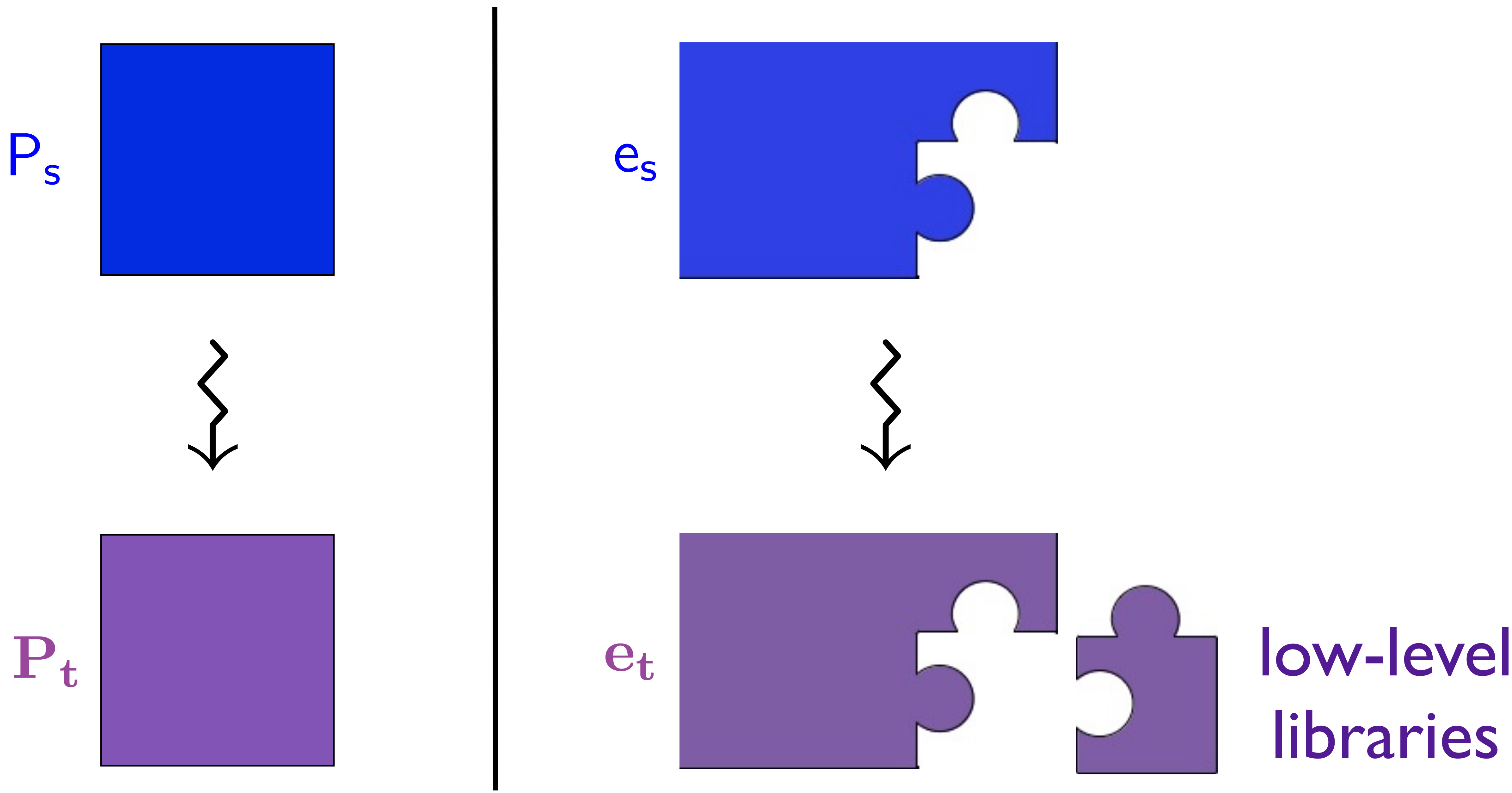

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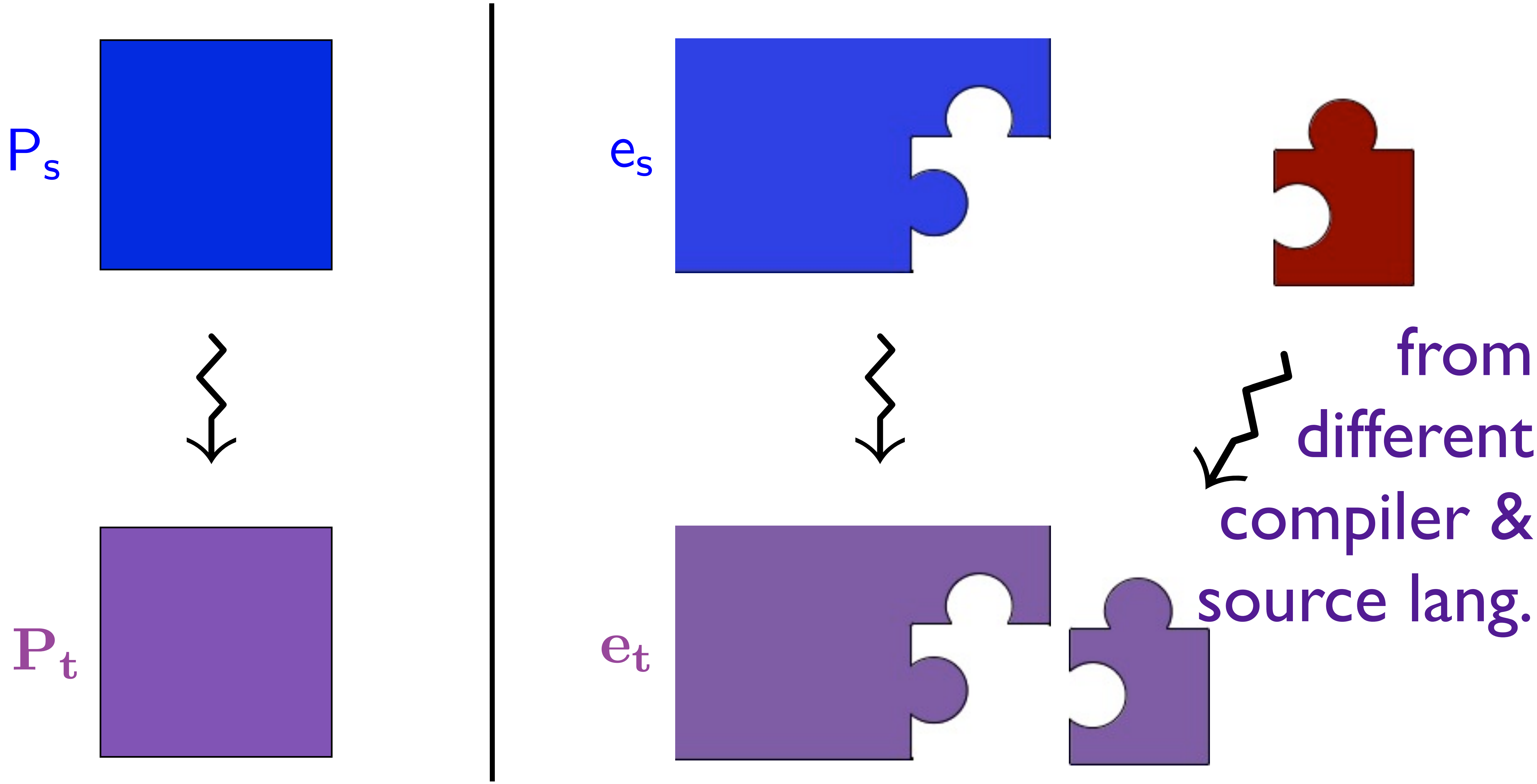


Why Whole Programs?

$$s \leadsto t \quad \Longrightarrow \quad s \approx t$$

expressed how?

Why Whole Programs?

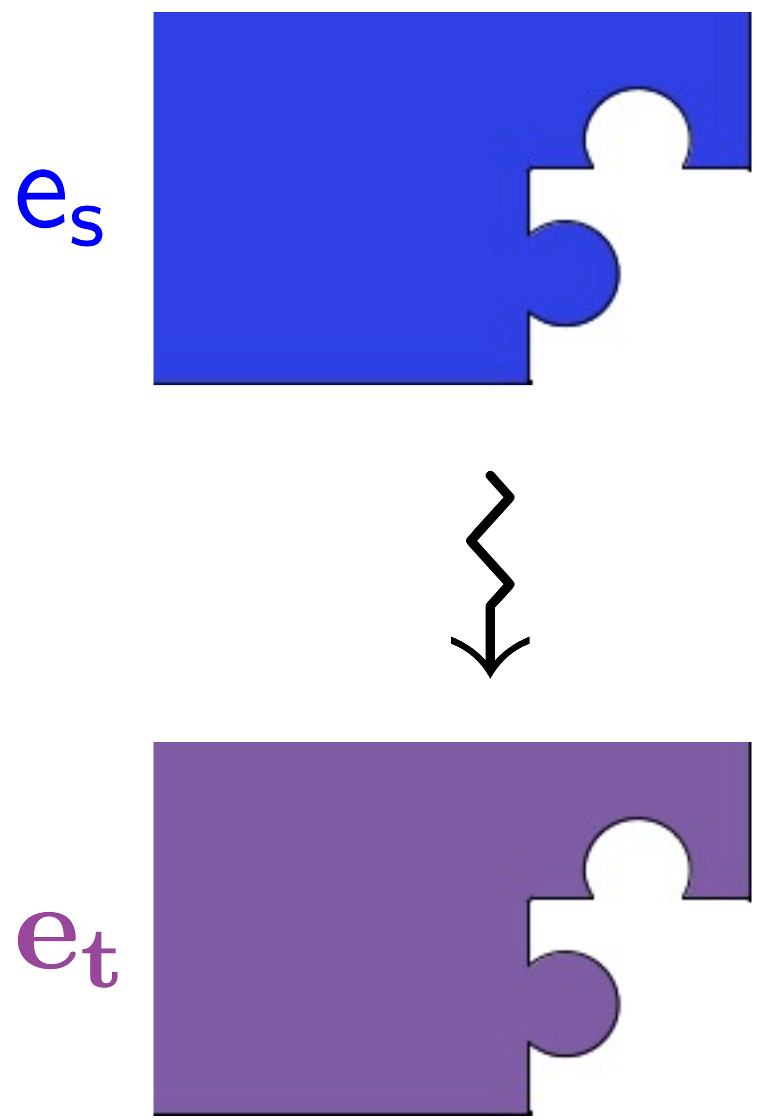
$$P_s \rightsquigarrow P_t \quad \Longrightarrow \quad P_s \approx P_t$$

expressed how?

CompCert

$$\begin{array}{ccccccccc}
P_s & \longmapsto & \ldots & \longmapsto & P_s^i & \longmapsto & P_s^{i+1} & \longmapsto & \ldots \\
| & & & & | & & \backslash & & \\
P_t & \longmapsto & \ldots & \longmapsto & P_t^j & \longmapsto^* & P_t^{j+n} & \longmapsto & \ldots
\end{array}$$

Correct Compilation of Components?



$e_S \approx e_T$

expressed how?

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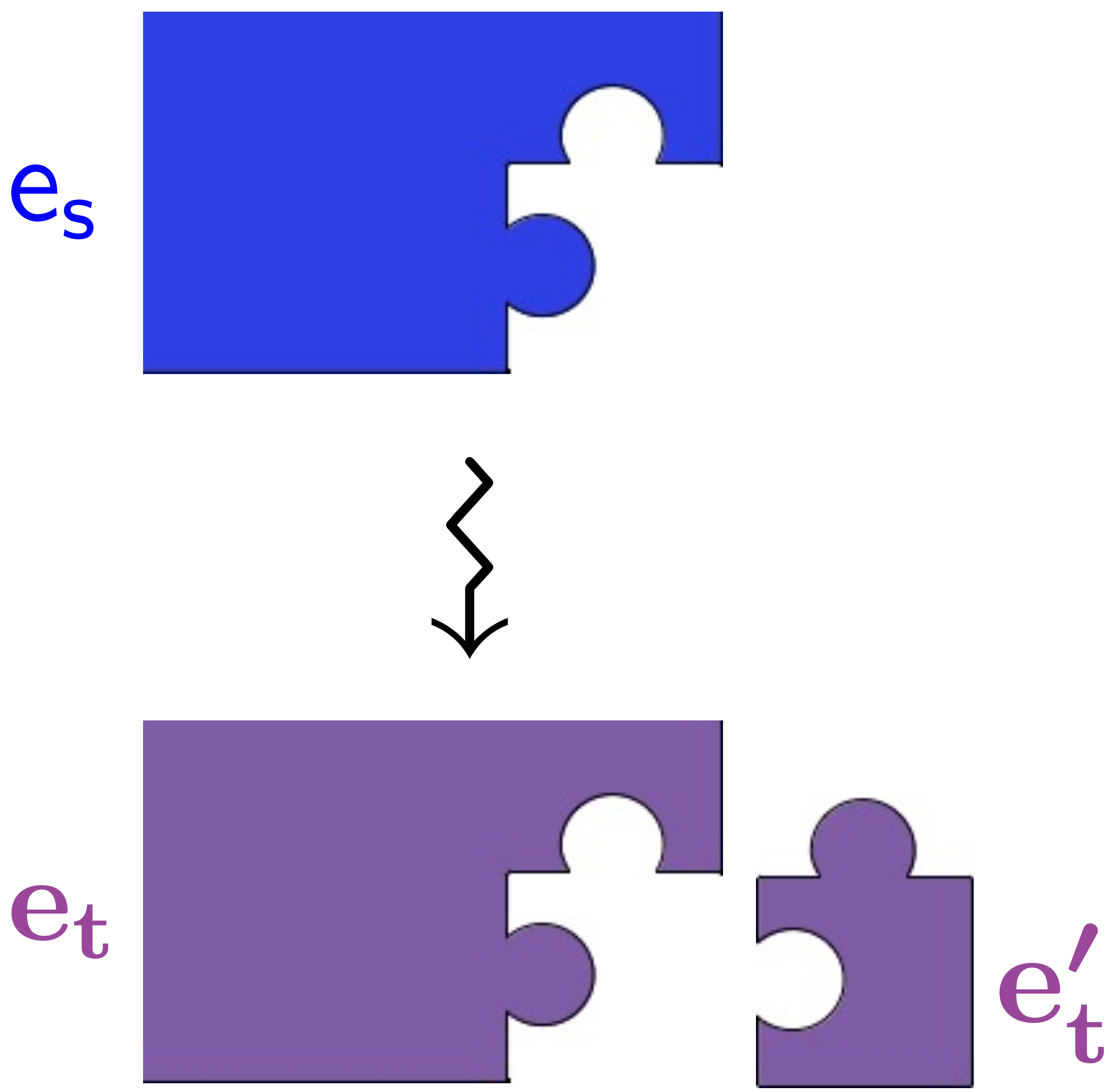

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↑

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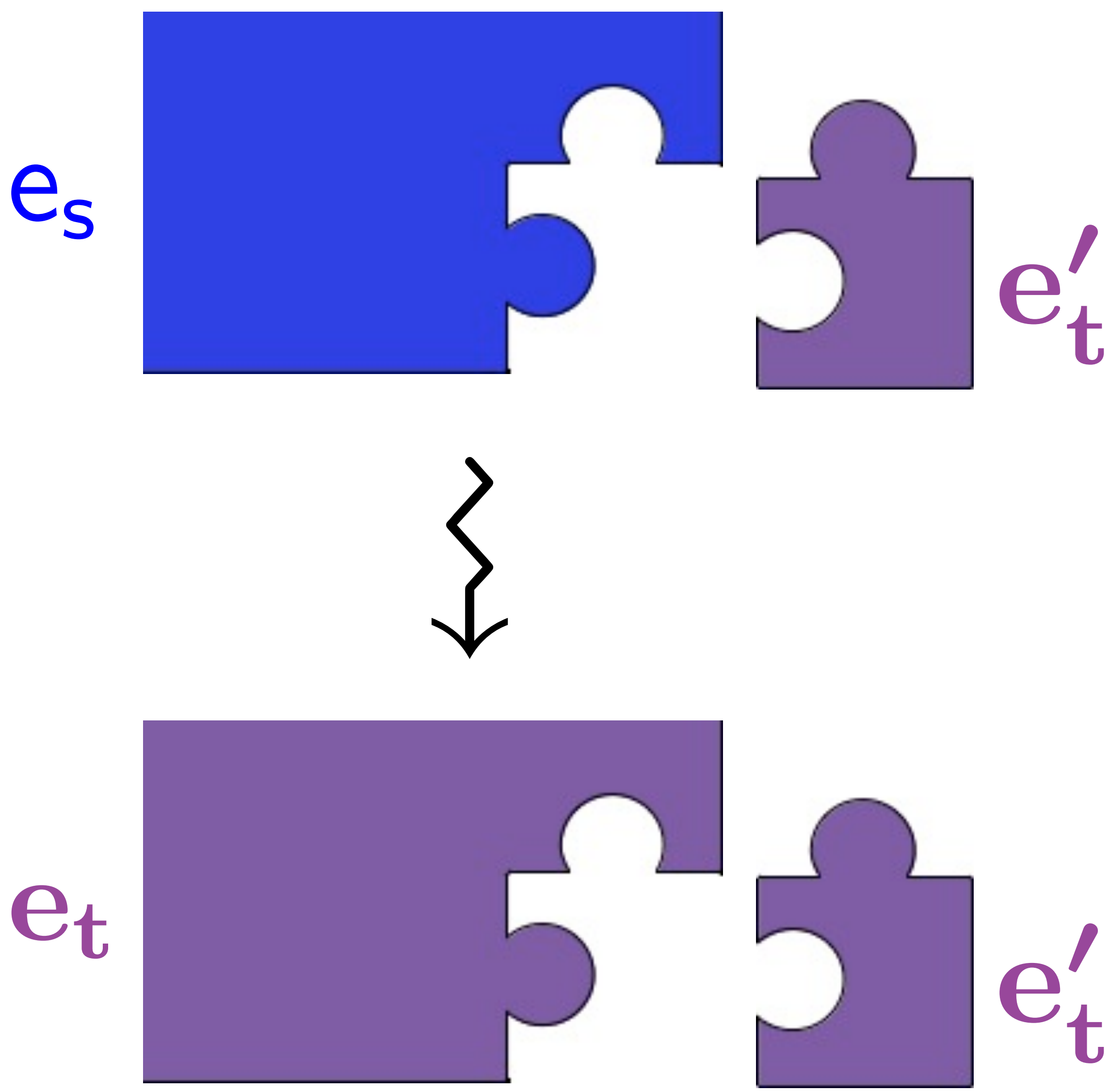

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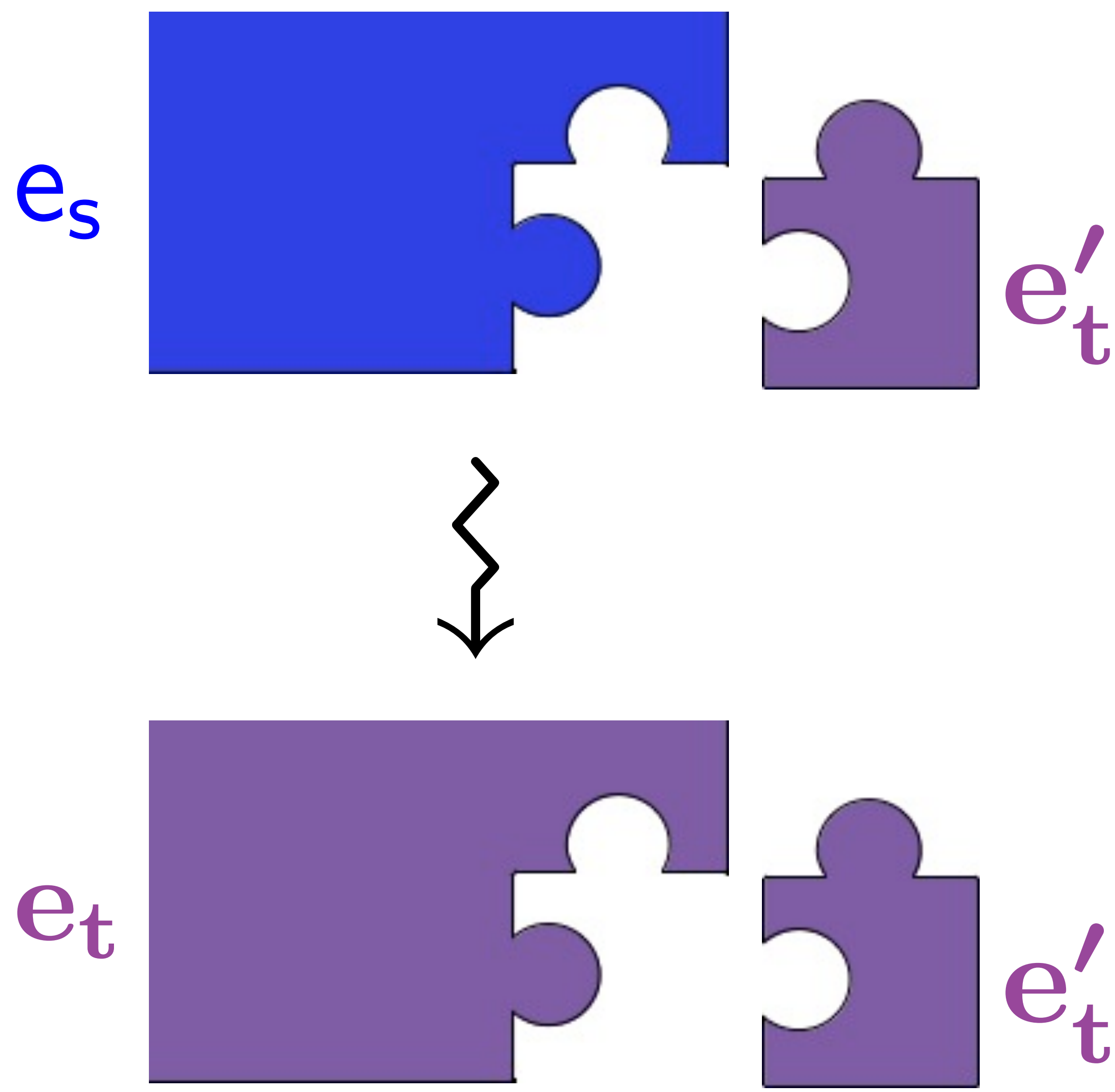

Need a semantics of source-target interoperability:

$\mathcal{ST}\, e_t \qquad \mathcal{TS}\, e_s$

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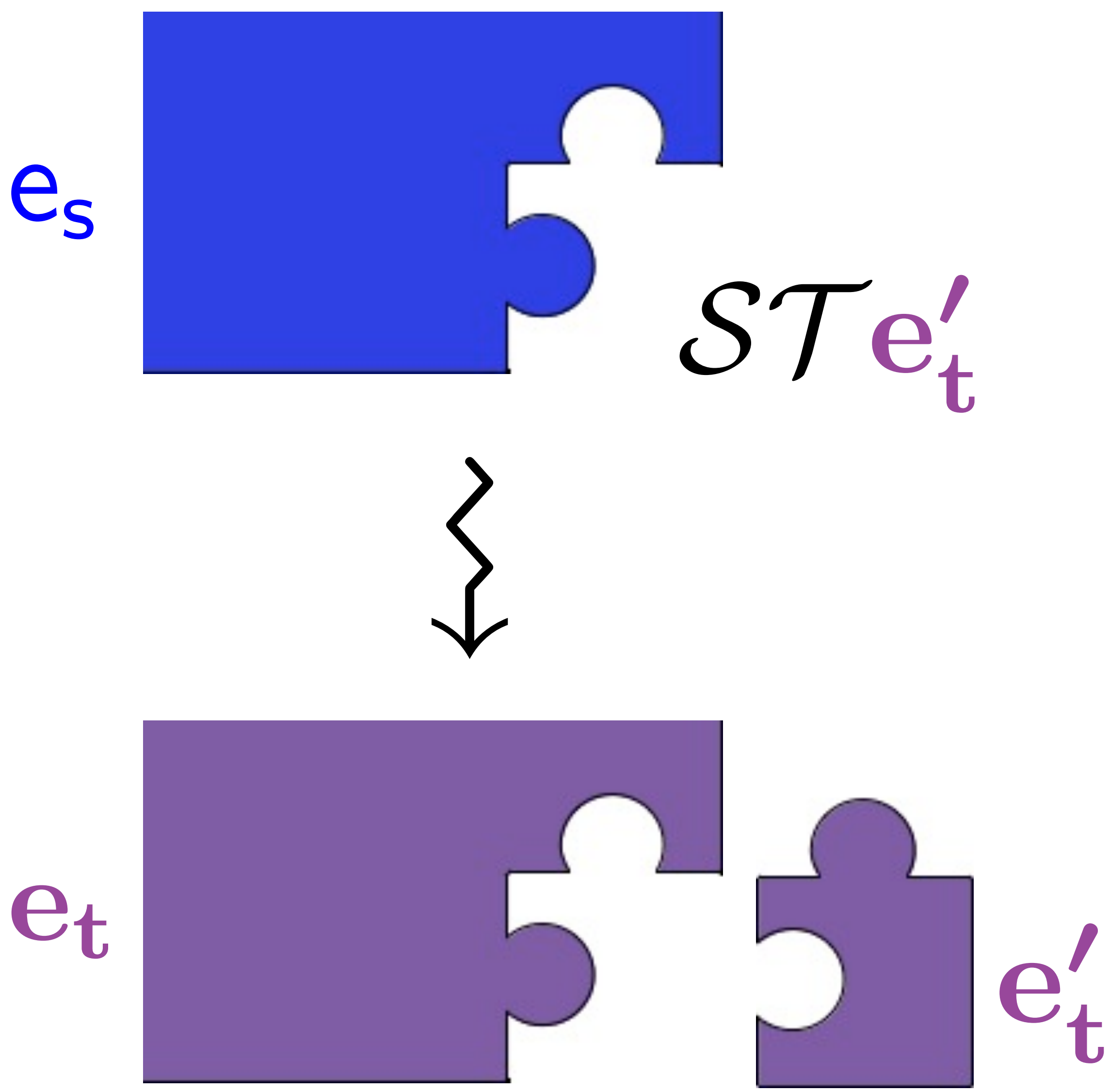

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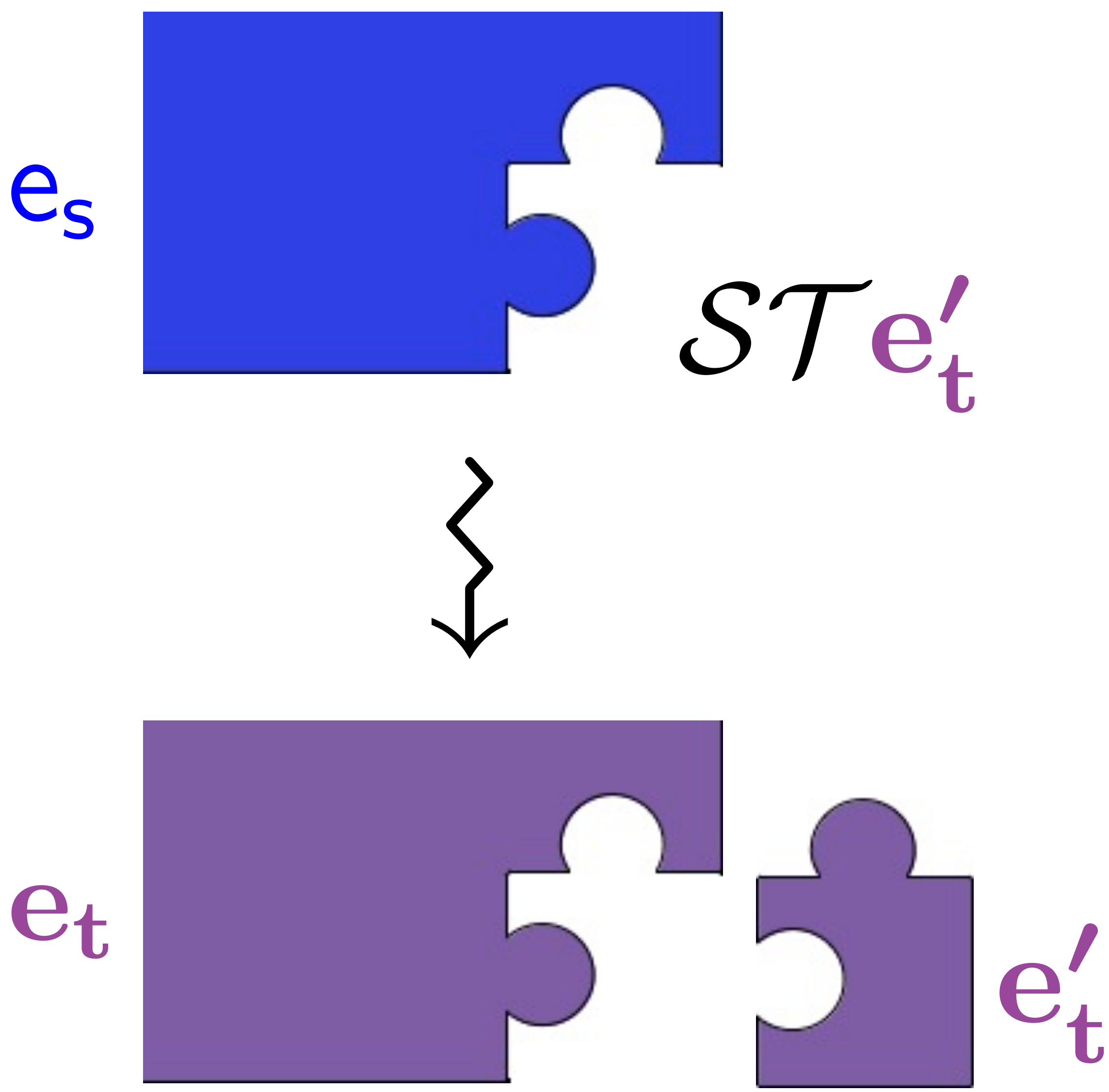
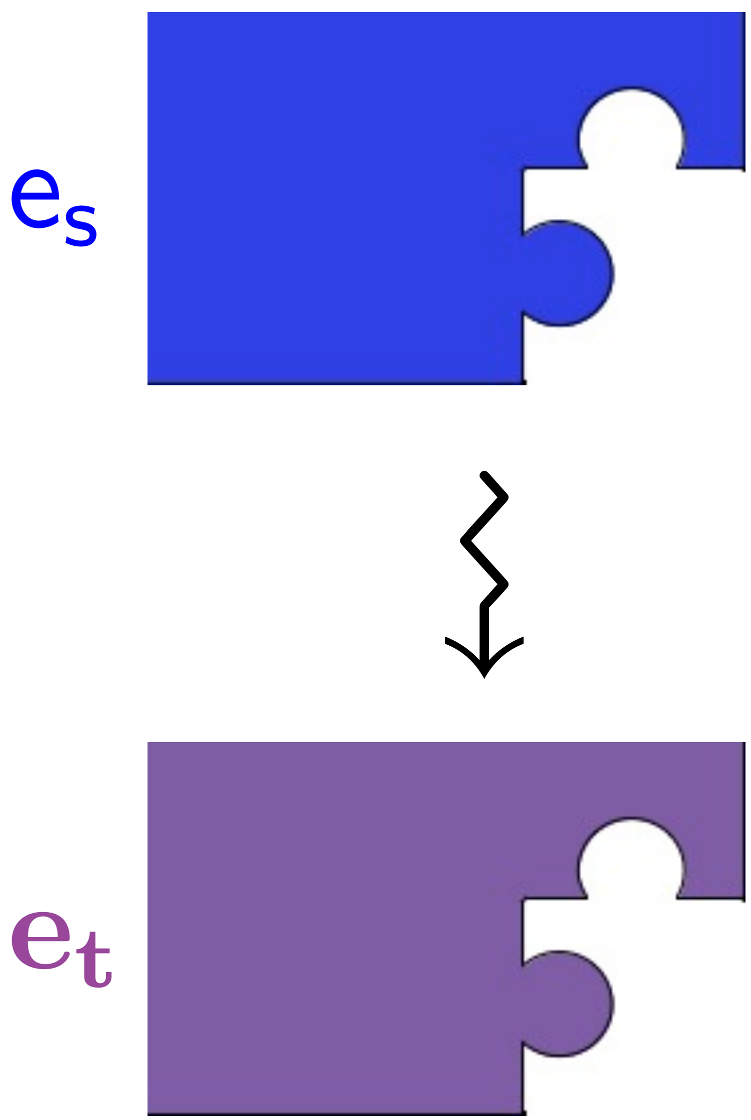


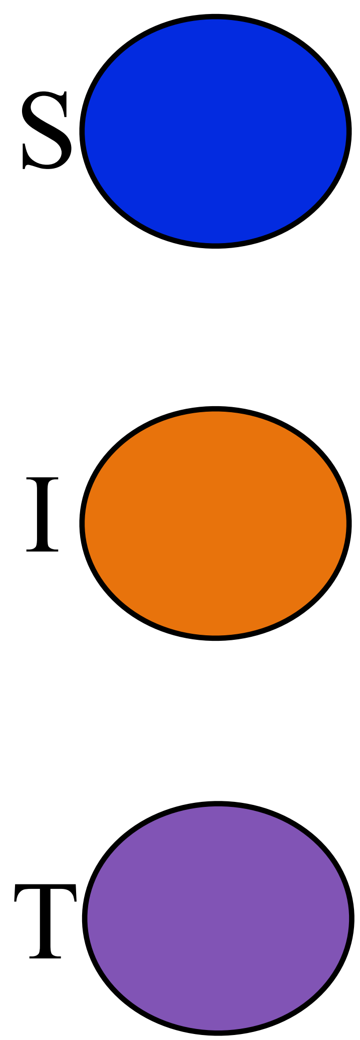
$$\mathcal{TS}(e_s\ (\mathcal{ST}\,e_t')) \approx^{ctx} e_t\ e_t'$$

Correct Compilation of Components



$$\mathbf{e_S} \approx \mathbf{e_T} \stackrel{\text{def}}{=} \mathbf{e_S} \approx^{ctx} \mathcal{S}\mathcal{T}\mathbf{e_T}$$

Our Approach (multi-pass compiler)



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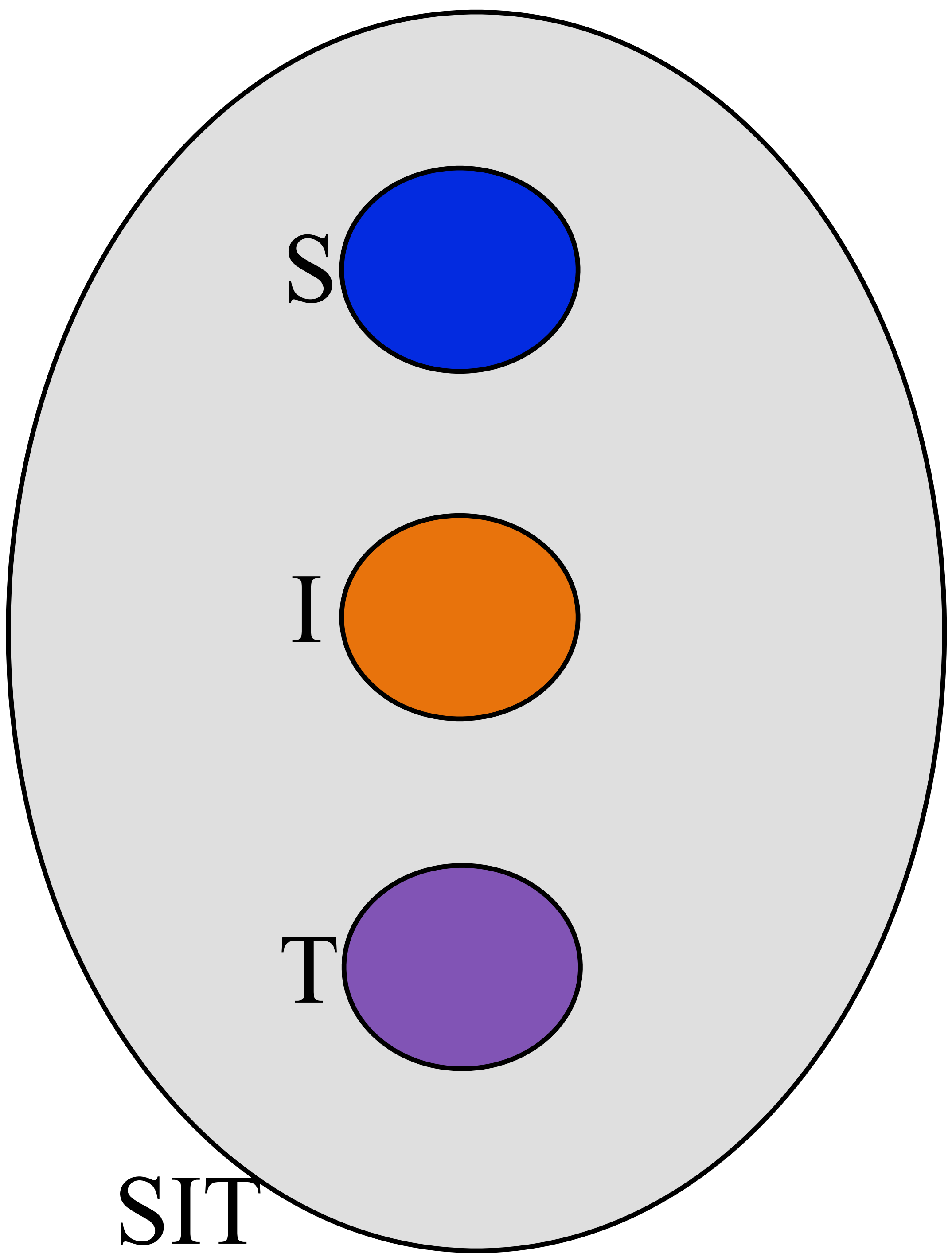

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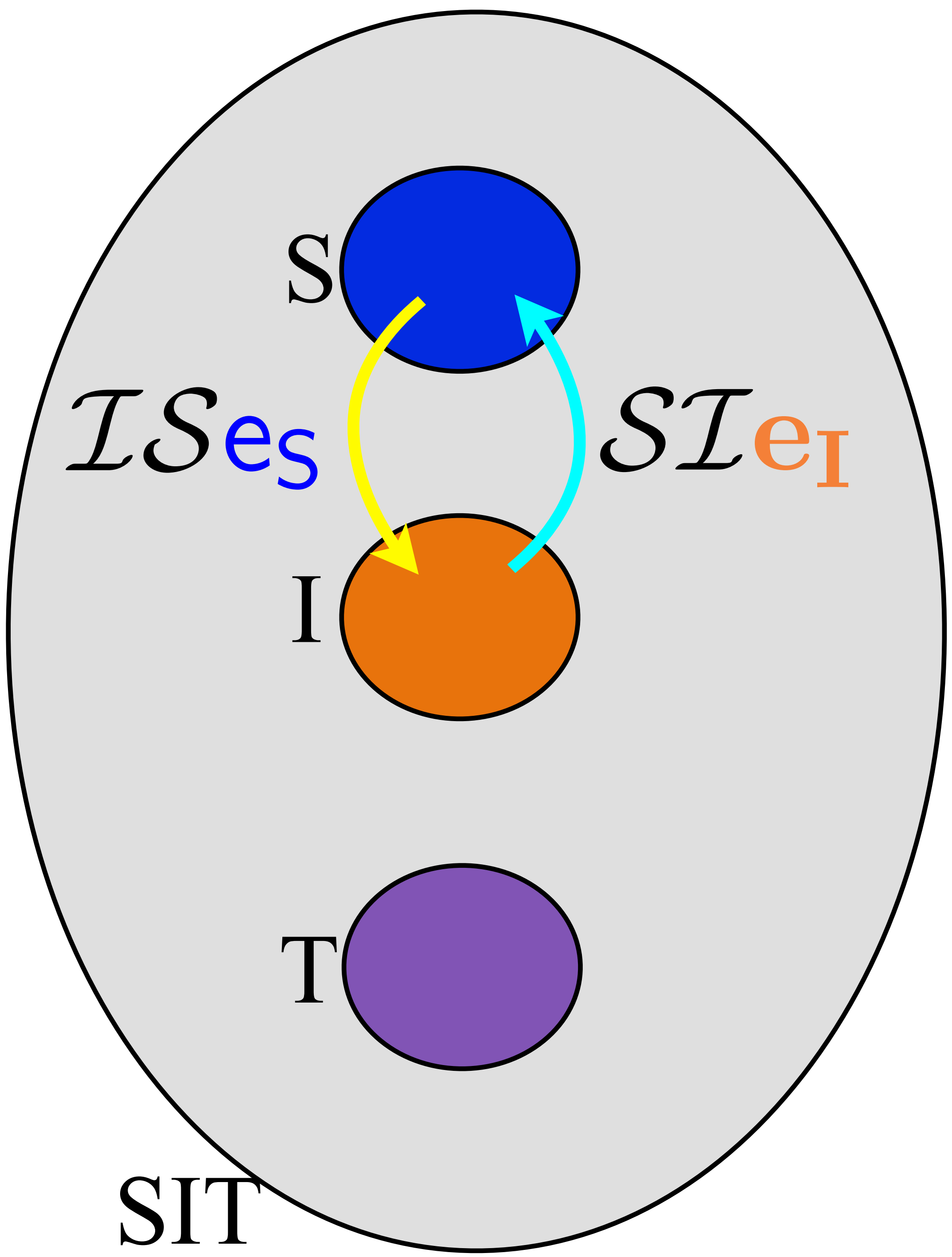

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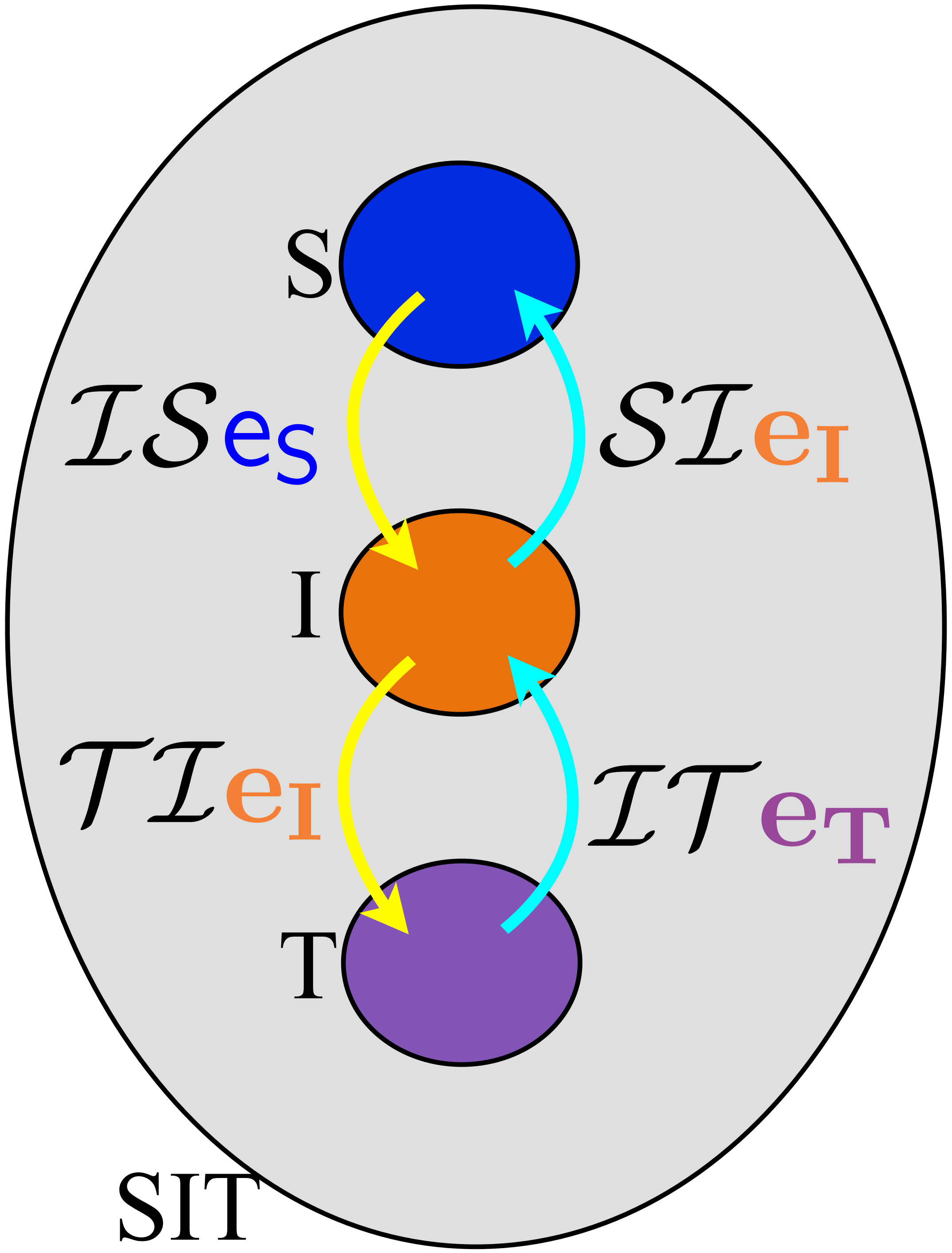

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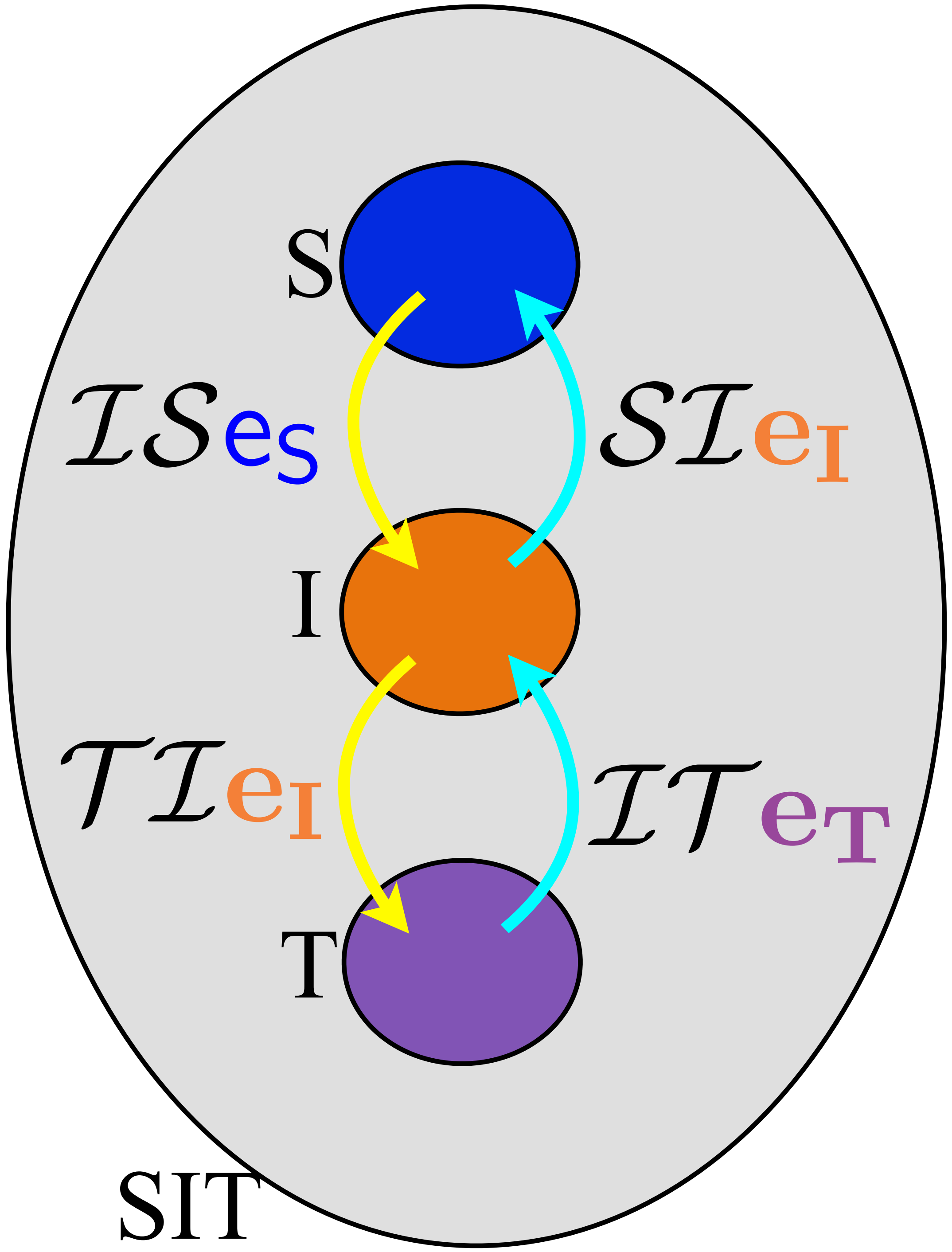
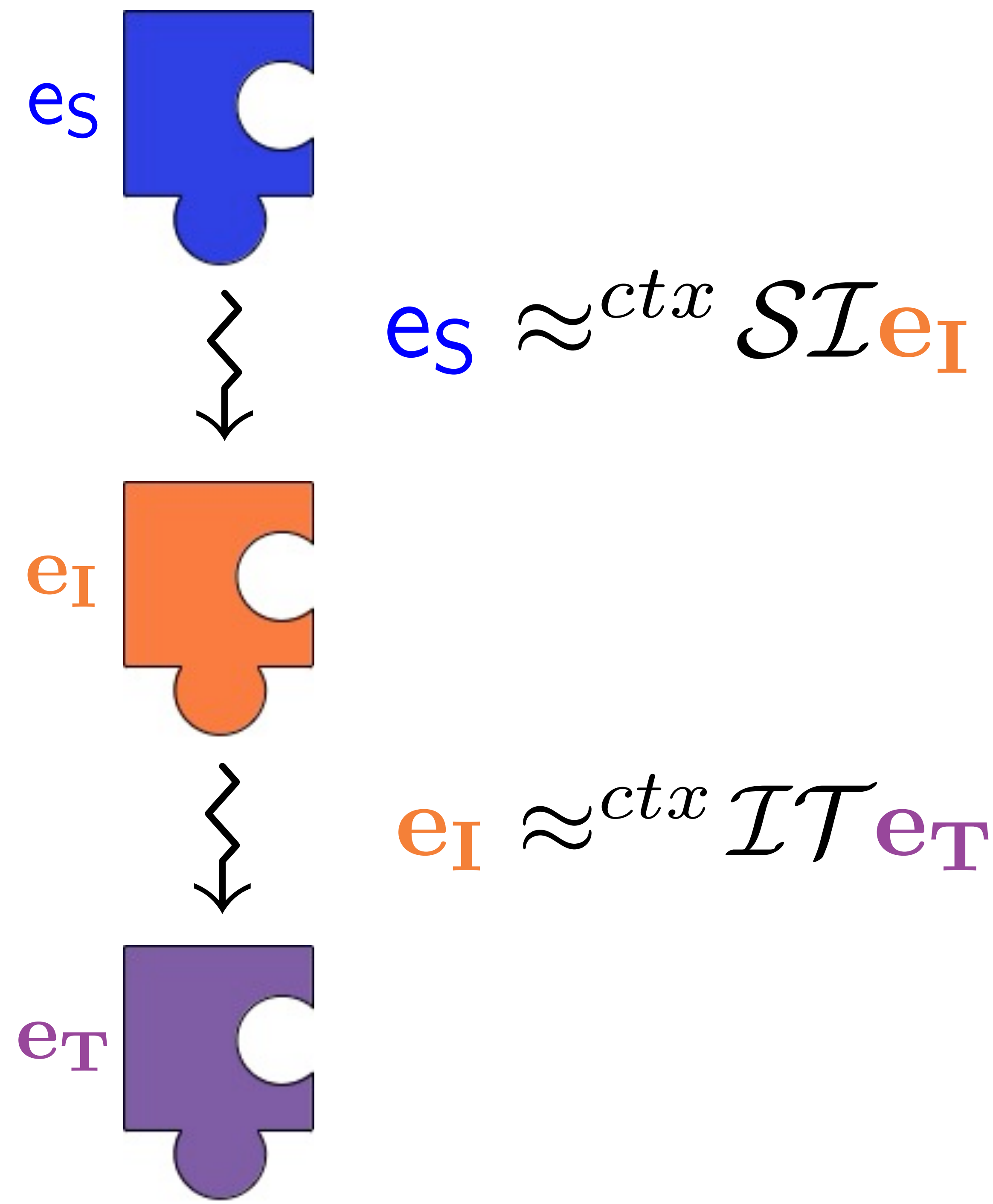
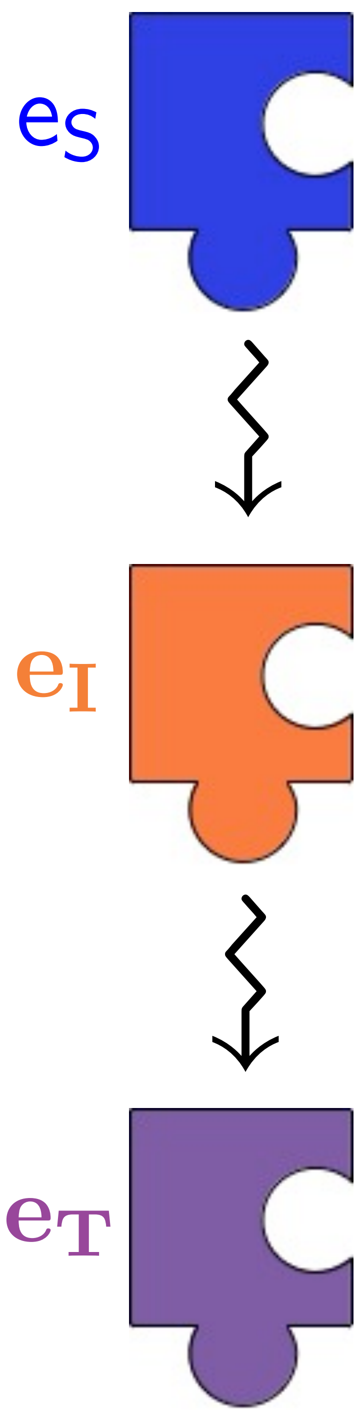


Compiler Correctness



Our Approach

Compiler Correctness

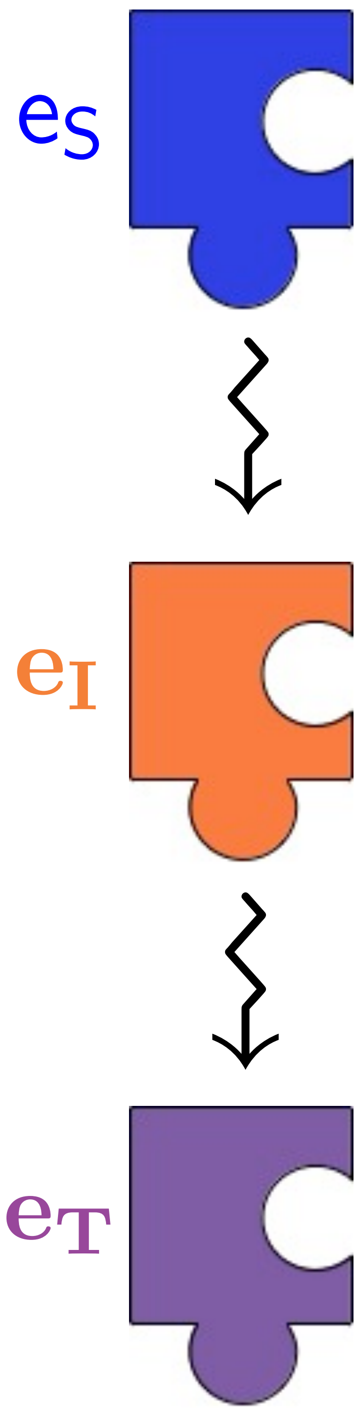


$$e_S \approx^{ctx}_{\mathcal{SI}} e_I$$

$$e_I \approx^{ctx}_{\mathcal{IT}} e_T$$

Our Approach

Compiler Correctness



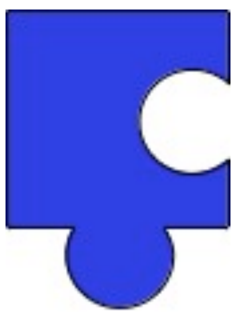
$$e_S \approx^{ctx} \mathcal{SI}\mathbf{e_I}$$

$$\mathcal{SI}\mathbf{e_I} \approx^{ctx} \mathcal{SI}(\mathcal{IT}\mathbf{e_T})$$

Our Approach

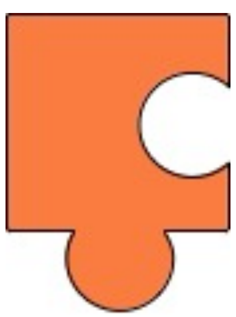
Compiler Correctness

e_S



$$e_S \approx^{ctx} \mathcal{S}\mathcal{I}\, e_I$$

e_I

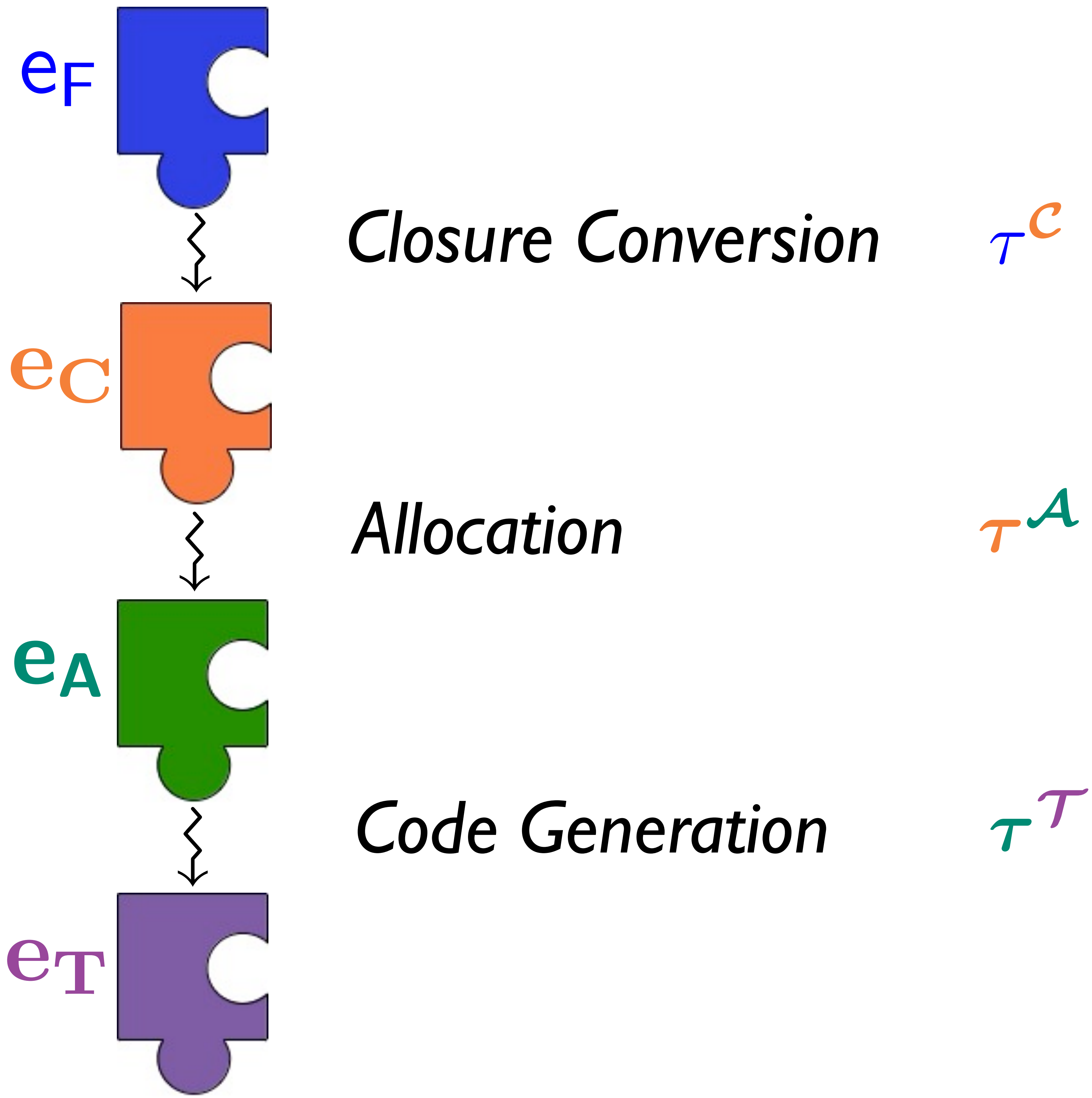


$$\mathcal{S}\mathcal{I}\, e_I \approx^{ctx} \mathcal{S}\mathcal{I}(\mathcal{I}\mathcal{T}\, e_T)$$

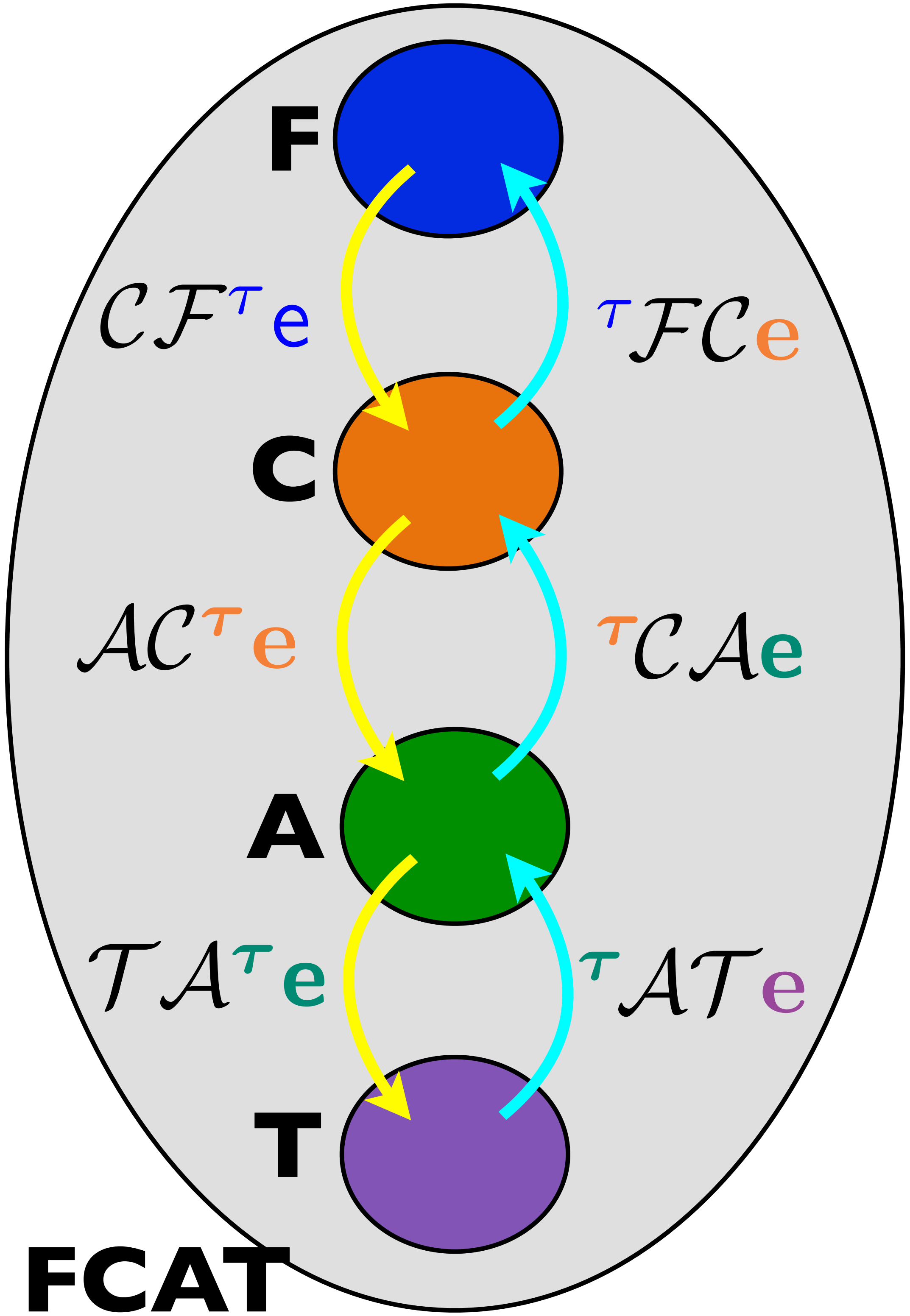
e_T

$$e_S \approx^{ctx} \mathcal{S}\mathcal{I}\mathcal{T}\, e_T$$

Our Compiler: System F to TAL



Combined language **FCAT**



- Boundaries mediate between
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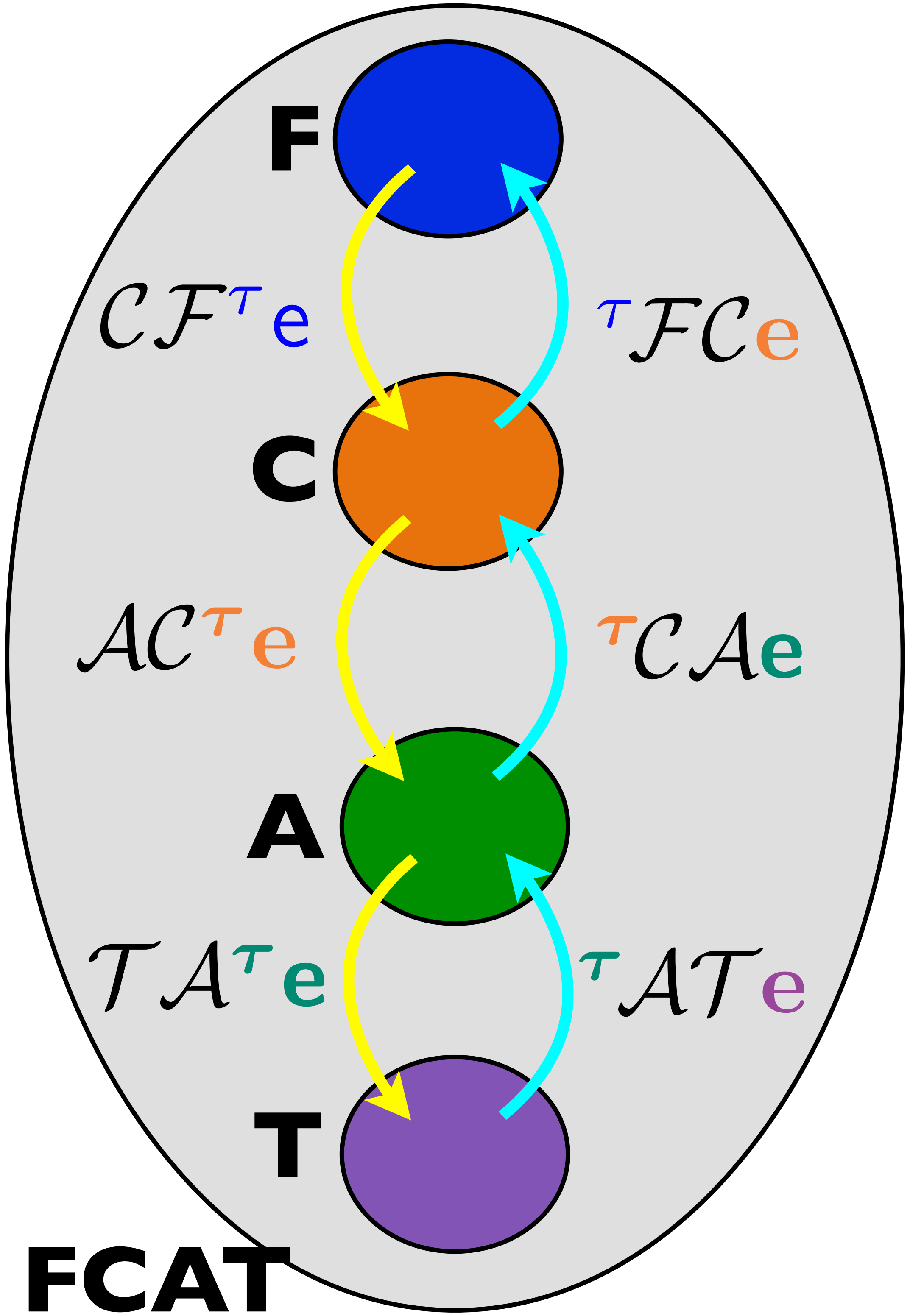

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- Operational semantics

$$\mathcal{CF}^{\tau}\mathsf{e} \longmapsto^{*} \mathcal{CF}^{\tau}\mathsf{v} \longmapsto \mathbf{v}$$

$$^{\tau}\mathcal{FC}\mathbf{e} \longmapsto^{*} {}^{\tau}\mathcal{FC}\mathbf{v} \longmapsto \mathsf{v}$$

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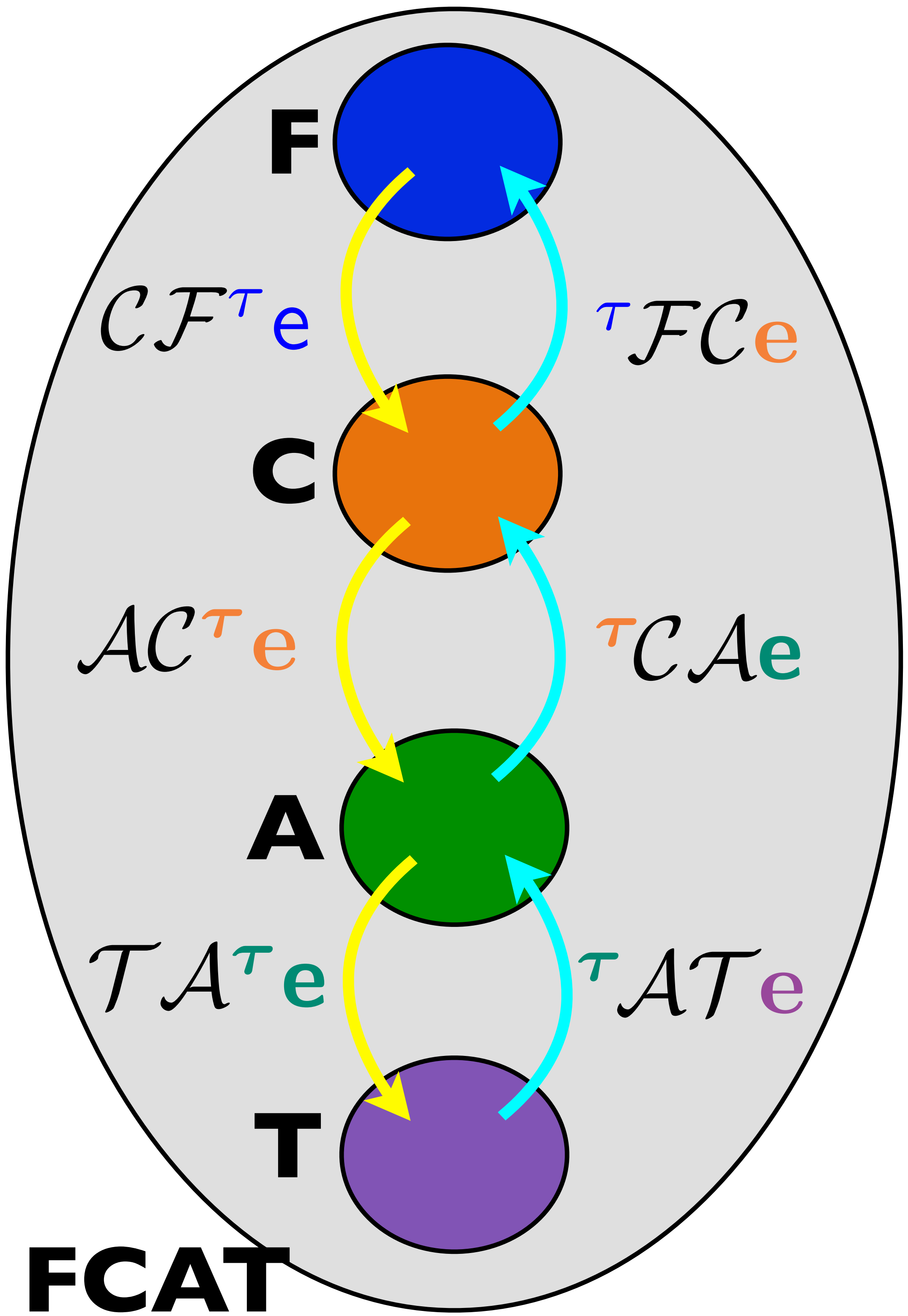

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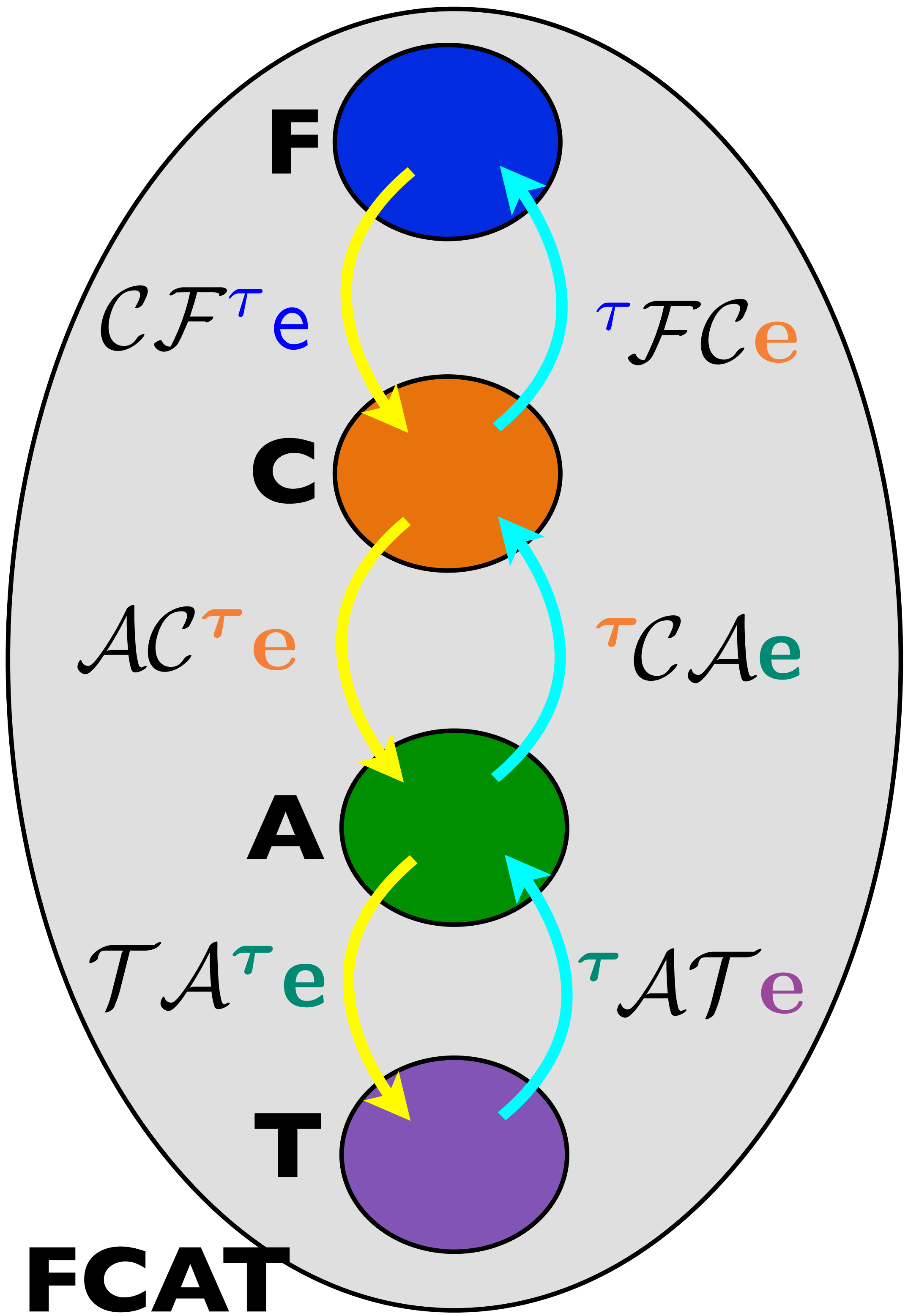
- Boundary cancellation

$$^{\tau}\mathcal{FC}\mathcal{CF}^{\tau}\mathsf{e} \approx^{ctx} \mathsf{e} : \tau$$

$$\mathcal{CF}^{\tau}\,{}^{\tau}\mathcal{FC}\mathbf{e} \approx^{ctx} \mathbf{e} : \tau^{\mathcal{C}}$$

Challenges / Roadmap for rest of talk



F+C: Interoperability semantics with type abstraction in both languages

C+A: Interoperability when compiler pass allocates code & tuples on heap

A+T: What is e? What is $\mathbf{v}$? How to define contextual equiv. for TAL *components*? How to define logical relation?

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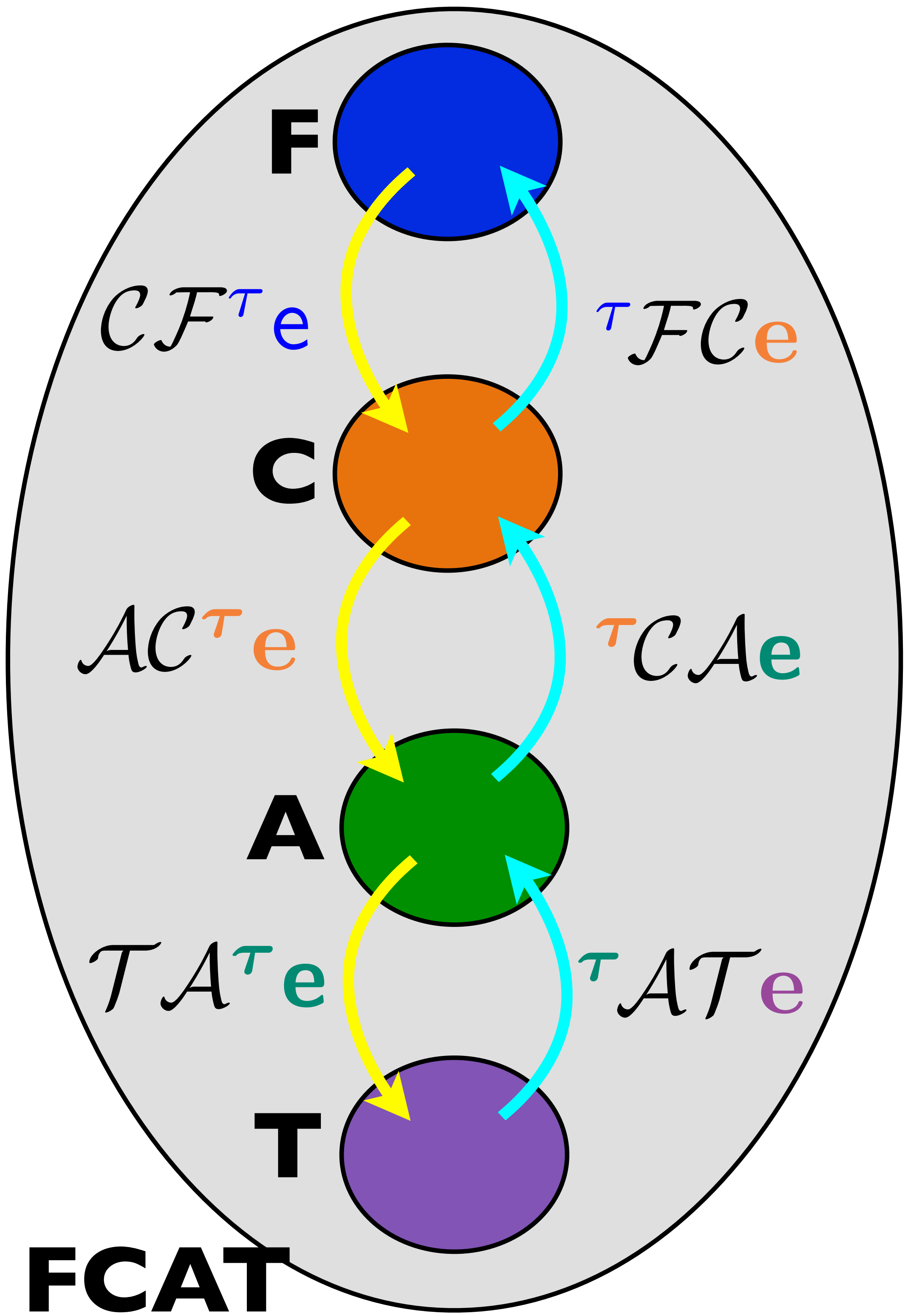

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Abstract Types & Interoperability

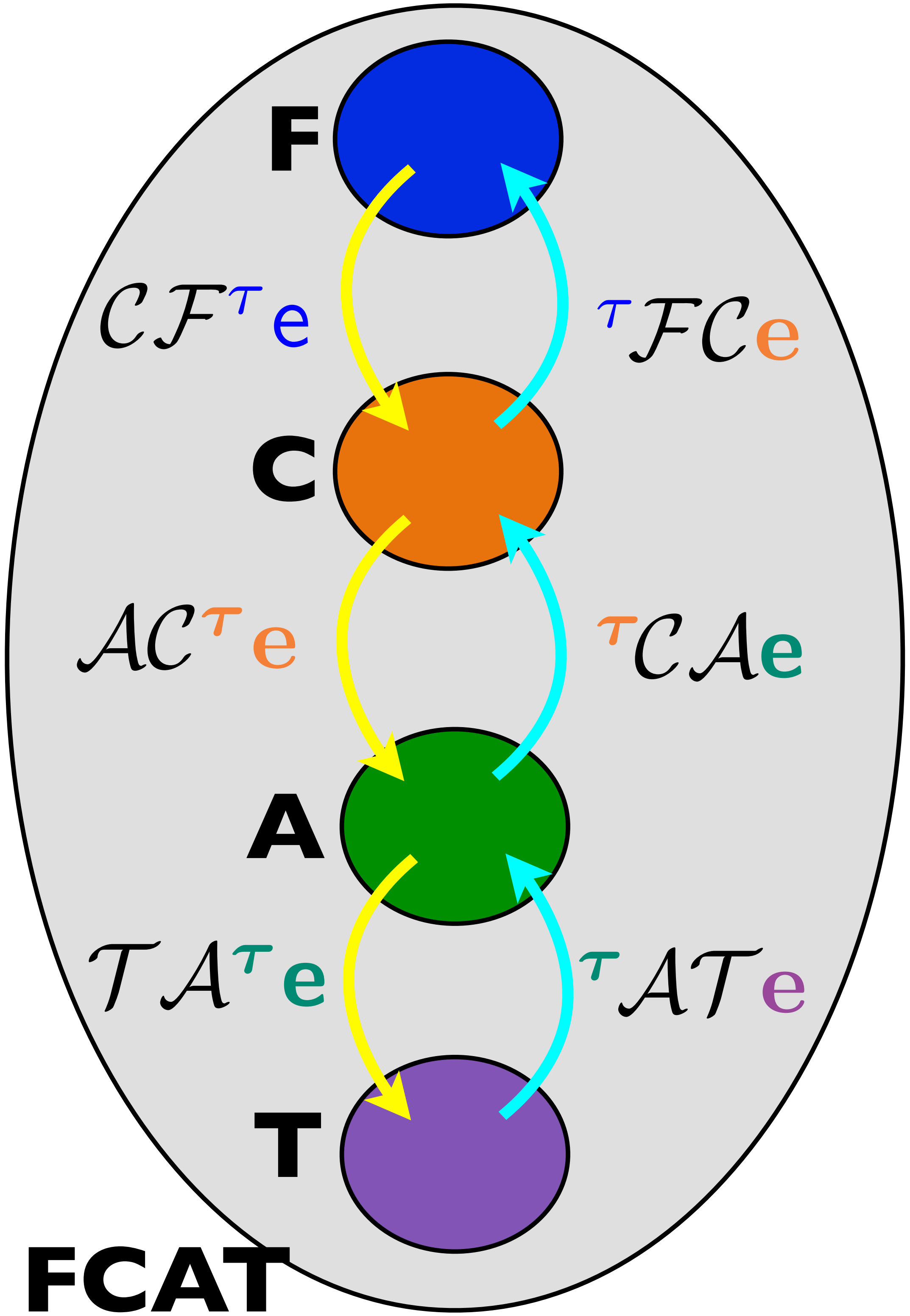
Add new type $\mathsf{L}\langle\boldsymbol{\tau}\rangle$ & new value form ${}^{\mathsf{L}\langle\boldsymbol{\tau}\rangle}\mathcal{FC}\mathbf{v}$

Add new type $\lceil\alpha\rceil$ & define $\lceil\alpha\rceil[\tau/\alpha] = \tau^{\langle\mathcal{C}\rangle}$

Requires novel admissibility relations in logical relation.

(draft paper: www.ccs.neu.edu/home/amal/voc.pdf)

Challenges / Roadmap



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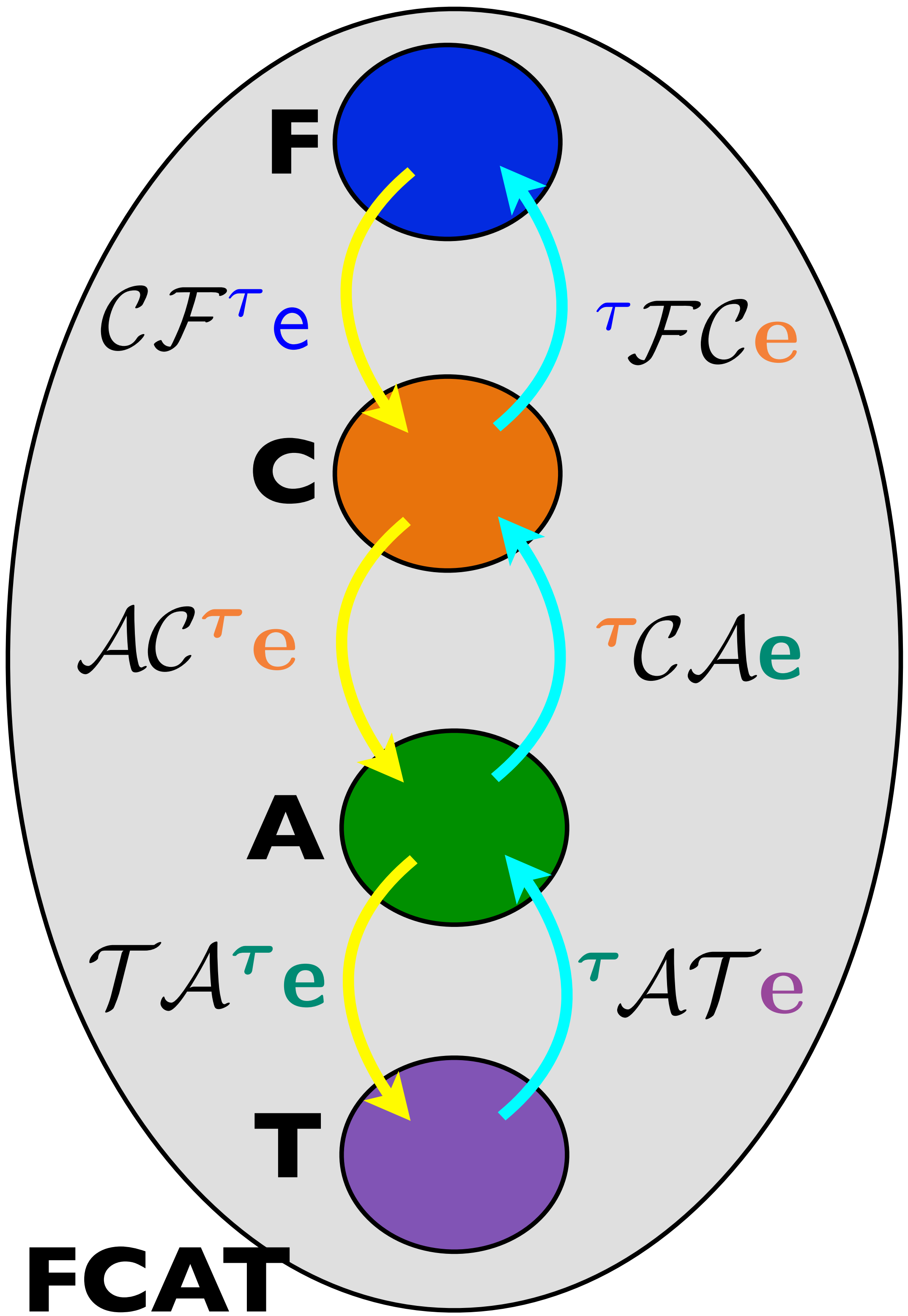

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A

$$\tau ::= \alpha \mid \mathsf{unit} \mid \mathsf{int} \mid \exists\alpha.\tau \mid \mu\alpha.\tau \mid \mathsf{box}\,\psi$$

$$\psi ::= \forall[\overline{\alpha}].(\overline{\tau}) \to \tau \mid \langle \tau, \ldots, \tau \rangle$$

$$\mathsf{e} ::= (\mathsf{t}, \mathsf{H}) \mid \mathsf{t}$$

$$\mathsf{t} ::= \mathsf{x} \mid () \mid \mathsf{n} \mid \mathsf{t\ p\ t} \mid \mathsf{if0\ t\ t\ t} \mid \ell \mid \mathsf{t}\,[]\,\bar{\mathsf{t}} \mid \mathsf{t}[\tau]$$
$$\mid \mathsf{pack}\langle \tau, \mathsf{t} \rangle\ \mathsf{as}\ \exists\alpha.\tau \mid \mathsf{unpack}\ \langle \alpha, \mathsf{x} \rangle = \mathsf{t\ in\ t} \mid \mathsf{fold}_{\mu\alpha.\tau}\ \mathsf{t}$$
$$\mid \mathsf{unfold\ t} \mid \mathsf{balloc}\ \langle \bar{\mathsf{t}} \rangle \mid \mathsf{read}[\mathsf{i}]\ \mathsf{t}$$

$$\mathsf{p} ::= + \mid - \mid *$$

$$\mathsf{v} ::= () \mid \mathsf{n} \mid \mathsf{pack}\langle \tau, \mathsf{v} \rangle\ \mathsf{as}\ \exists\alpha.\tau \mid \mathsf{fold}_{\mu\alpha.\tau}\ \mathsf{v} \mid \ell \mid \mathsf{v}[\tau]$$

$$\mathsf{H} ::= \cdot \mid \mathsf{H}, \ell \mapsto \mathsf{h}$$

$$\mathsf{h} ::= \lambda[\overline{\alpha}]\,(\overline{\mathsf{x} : \tau}).\mathsf{t} \mid \langle \mathsf{v}, \ldots, \mathsf{v} \rangle$$

$\boxed{\langle \mathsf{H} \mid \mathsf{e} \rangle \longmapsto \langle \mathsf{H}' \mid \mathsf{e}' \rangle}$ Reduction Relation (selected cases)

$$\langle \mathsf{H} \mid (\mathsf{t}, \mathsf{H}') \rangle \longmapsto \langle (\mathsf{H}, \mathsf{H}') \mid \mathsf{t} \rangle \qquad \mathrm{dom}(\mathsf{H}) \cap \mathrm{dom}(\mathsf{H}') = \emptyset$$

$$\langle \mathsf{H} \mid \mathsf{E}[\ell\ [\overline{\tau'}]\ \overline{\mathsf{v}}] \rangle \longmapsto \langle \mathsf{H} \mid \mathsf{E}[\mathsf{t}[\overline{\tau'}/\overline{\alpha}][\overline{\mathsf{v}}/\overline{\mathsf{x}}]] \rangle \quad \mathsf{H}(\ell) = \lambda[\overline{\alpha}]\,(\overline{\mathsf{x} : \tau}).\mathsf{t}$$

T

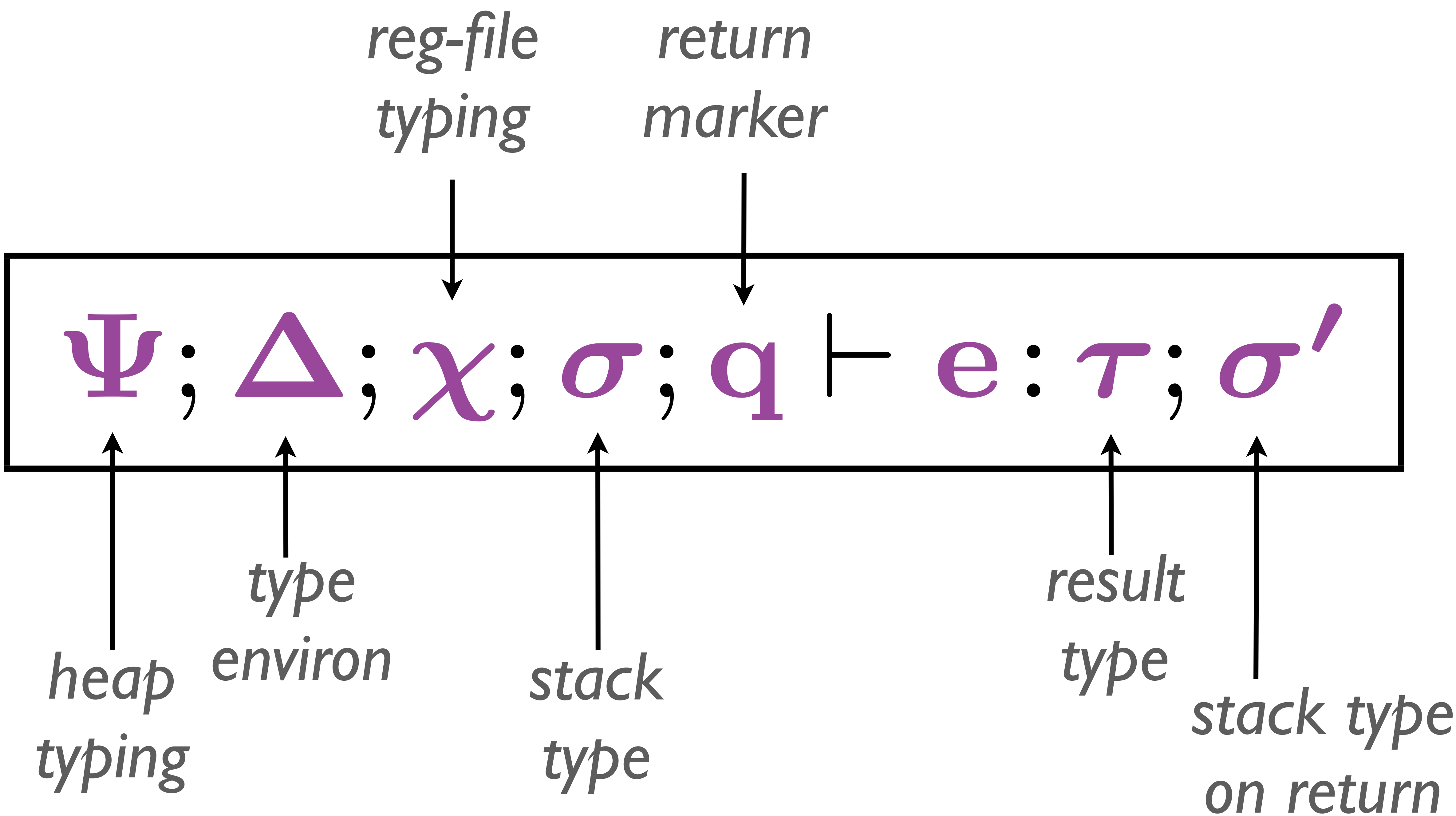
$$
\begin{array}{llll}
\tau & ::= & \alpha \mid \mathbf{unit} \mid \mathbf{int} \mid \exists\alpha.\tau \mid \mu\alpha.\tau & \textit{Type} \\
 & \mid & \mathbf{ref}\ \langle\tau,\ldots,\tau\rangle \mid \mathbf{box}\ \psi & \\
\psi & ::= & \forall[\Delta].\{\chi;\sigma\}^{\mathrm{q}} \mid \langle\tau,\ldots,\tau\rangle & \textit{Heap value type} \\
\chi & ::= & \cdot \mid \chi, \mathrm{r}:\tau & \textit{Register file type} \\
\sigma & ::= & \zeta \mid \bullet \mid \tau :: \sigma & \textit{Stack type} \\
\mathrm{q} & ::= & \epsilon \mid \mathrm{r} \mid \mathrm{i} \mid \mathrm{end}[\tau;\sigma] & \textit{Return marker} \\
\Delta & ::= & \cdot \mid \Delta,\alpha \mid \Delta,\zeta \mid \Delta,\epsilon & \textit{Type variable environment} \\
\omega & ::= & \tau \mid \sigma \mid \mathrm{q} & \textit{Instantiation of type variable} \\
\mathrm{r} & ::= & \mathtt{r1} \mid \mathtt{r2} \mid \cdots \mid \mathtt{r7} \mid \mathtt{ra} & \textit{Register} \\
\mathrm{h} & ::= & \mathrm{code}[\Delta]\{\chi;\sigma\}^{\mathrm{q}}.\mathrm{I} \mid \langle\mathrm{w},\ldots,\mathrm{w}\rangle & \textit{Heap value} \\
\mathrm{w} & ::= & () \mid \mathrm{n} \mid \ell \mid \mathrm{pack}\langle\tau,\mathrm{w}\rangle\ \mathrm{as}\ \exists\alpha.\tau & \textit{Word value} \\
 & \mid & \mathrm{fold}_{\mu\alpha.\tau}\ \mathrm{w} \mid \mathrm{w}[\omega] & \\
\mathrm{u} & ::= & \mathrm{w} \mid \mathrm{r} \mid \mathrm{pack}\langle\tau,\mathrm{u}\rangle\ \mathrm{as}\ \exists\alpha.\tau & \textit{Small value} \\
 & \mid & \mathrm{fold}_{\mu\alpha.\tau}\ \mathrm{u} \mid \mathrm{u}[\omega] & \\
\mathrm{I} & ::= & \iota;\mathrm{I} \mid \mathtt{jmp}\ \mathrm{u} \mid \mathtt{ret}\ \mathrm{q},\mathrm{r} & \textit{Instruction sequence}
\end{array}
$$

T

$$
\begin{array}{rcll}
\iota & ::= & \mathtt{aop}\ \mathbf{r_d}, \mathbf{r_s}, \mathbf{u} \mid \mathtt{bnz}\ \mathbf{r}, \mathbf{u} \mid \mathtt{mv}\ \mathbf{r_d}, \mathbf{u} & \textit{Instruction} \\
& \mid & \mathtt{ralloc}\ \mathbf{r_d}, \mathbf{n} \mid \mathtt{balloc}\ \mathbf{r_d}, \mathbf{n} \mid \mathtt{ld}\ \mathbf{r_d}, \mathbf{r_s}[\mathbf{i}] \mid \mathtt{st}\ \mathbf{r_d}[\mathbf{i}], \mathbf{r_s} & \\
& \mid & \mathtt{unpack}\ \langle \alpha, \mathbf{r_d} \rangle\ \mathbf{u} \mid \mathtt{unfold}\ \mathbf{r_d}, \mathbf{u} \mid \mathtt{salloc}\ \mathbf{n} \mid \mathtt{sfree}\ \mathbf{n} & \\
& \mid & \mathtt{sld}\ \mathbf{r_d}, \mathbf{i} \mid \mathtt{sst}\ \mathbf{i}, \mathbf{r_s} & \\
\mathbf{aop} & ::= & \mathtt{add} \mid \mathtt{sub} \mid \mathtt{mult} & \textit{Arithmetic operation} \\
\mathbf{e} & ::= & (\mathbf{I}, \mathbf{H}) \mid \mathbf{I} & \textit{Component} \\
\mathbf{v} & ::= & \mathtt{ret}\ \mathbf{q}, \mathbf{r} & \textit{Term value} \\
\mathbf{E} & ::= & (\mathbf{E_I}, \cdot) & \textit{Evaluation context} \\
\mathbf{E_I} & ::= & [\cdot] & \textit{Instruction evaluation context} \\
\mathbf{H} & ::= & \cdot \mid \mathbf{H}, \ell \mapsto \mathbf{h} & \textit{Heap or Heap fragment} \\
\mathbf{R} & ::= & \cdot \mid \mathbf{R}, \mathbf{r} \mapsto \mathbf{w} & \textit{Register file} \\
\mathbf{S} & ::= & \mathbf{nil} \mid \mathbf{w} :: \mathbf{S} & \textit{Stack} \\
\mathbf{M} & ::= & (\mathbf{H}, \mathbf{R}, \mathbf{S} : \sigma) & \textit{Memory}
\end{array}
$$

$$
\boxed{\langle \mathbf{M} \mid \mathbf{e} \rangle \longmapsto \langle \mathbf{M'} \mid \mathbf{e'} \rangle}
$$

Typing TAL Components



Well-typed Components in $\mathbf{T}$

$$\boxed{\Psi; \Delta; \chi; \sigma; \mathbf{q} \vdash \mathbf{e} : \tau; \sigma'}$$

$$\frac{\begin{array}{c}\Psi \vdash \mathbf{H} : \Psi_{\mathbf{e}} \qquad \text{boxheap}(\Psi_{\mathbf{e}}) \\ \text{ret-type}(\mathbf{q}, \chi, \sigma) = \tau; \sigma' \qquad (\Psi, \Psi_{\mathbf{e}}); \Delta; \chi; \sigma; \mathbf{q} \vdash \mathbf{I}\end{array}}{\Psi; \Delta; \chi; \sigma; \mathbf{q} \vdash (\mathbf{I}, \mathbf{H}) : \tau; \sigma'}$$

Well-typed Instruction Sequence

$$\boxed{\Psi; \Delta; \chi; \sigma; \mathrm{q} \vdash \mathrm{I}} \quad \text{where } \mathrm{q} \neq \epsilon$$

$$\frac{\Psi; \Delta; \chi; \sigma; \mathrm{q} \vdash \iota \Rightarrow \Delta'; \chi'; \sigma'; \mathrm{q}' \qquad \Psi; \Delta'; \chi'; \sigma'; \mathrm{q}' \vdash \mathrm{I}}{\Psi; \Delta; \chi; \sigma; \mathrm{q} \vdash \iota; \mathrm{I}}$$

$$\frac{\chi(\mathrm{r}) = \mathrm{box}\, \forall[].\{\mathrm{r}' : \tau; \sigma\}^{\mathrm{q}'} \qquad \chi(\mathrm{r}') = \tau}{\Psi; \Delta; \chi; \sigma; \mathrm{r} \vdash \mathtt{ret}\; \mathrm{r}, \mathrm{r}'}$$

$$\frac{\chi(\mathrm{r}) = \tau}{\Psi; \Delta; \chi; \sigma; \mathrm{end}[\tau; \sigma] \vdash \mathtt{ret}\; \mathrm{end}[\tau; \sigma], \mathrm{r}}$$

Jmp

To next code block within component:

$$\frac{\Psi; \Delta; \chi \vdash u : \mathrm{box}\, \forall[].\{\chi'; \sigma\}^{q} \qquad \Delta \vdash \chi \leq \chi'}{\Psi; \Delta; \chi; \sigma; q \vdash \mathtt{jmp}\, u}$$

Call subroutine:

- must protect current return addr, by storing it in tail part of stack that is parametrically hidden from subroutine

$$\frac{\begin{array}{c}\Psi; \Delta; \chi \vdash u : \mathrm{box}\, \forall[\zeta, \epsilon].\{\hat{\chi}; \hat{\sigma}\}^{\hat{q}} \qquad \text{ret-addr-type}(\hat{q}, \hat{\chi}, \hat{\sigma}) = \forall[].\{r : \tau; \hat{\sigma}'\}^{\epsilon} \\ \Delta \vdash \sigma_0 \qquad \Delta \vdash \forall[].\{\hat{\chi}[\sigma_0/\zeta][i{+}k{-}j/\epsilon]; \hat{\sigma}[\sigma_0/\zeta][i{+}k{-}j/\epsilon]\}^{\hat{q}} \qquad \Delta \vdash \chi \leq \hat{\chi}[\sigma_0/\zeta][i{+}k{-}j/\epsilon] \\ \sigma = \tau_0 :: \cdots :: \tau_j :: \sigma_0 \qquad \hat{\sigma} = \tau_0 :: \cdots :: \tau_j :: \zeta \qquad j < i \qquad \hat{\sigma}' = \tau_0' :: \cdots :: \tau_k' :: \zeta\end{array}}{\Psi; \Delta; \chi; \sigma; i \vdash \mathtt{jmp}\, u[\sigma_0, i{+}k{-}j]}$$

Instruction Typing

Instructions must not clobber return address:

$$\frac{\Psi; \Delta; \chi \vdash \mathbf{u} : \tau \qquad \mathbf{q} \neq \mathbf{r_d}}{\Psi; \Delta; \chi; \sigma; \mathbf{q} \vdash \texttt{mv}\ \mathbf{r_d}, \mathbf{u} \Rightarrow \Delta; \chi[\mathbf{r_d} : \tau]; \sigma; \mathbf{q}}$$

Can move return address elsewhere:

$$\frac{\Psi; \Delta; \chi \vdash \mathbf{u} : \tau}{\Psi; \Delta; \chi; \sigma; \mathbf{r_s} \vdash \texttt{mv}\ \mathbf{r_d}, \mathbf{r_s} \Rightarrow \Delta; \chi[\mathbf{r_d} : \tau]; \sigma; \mathbf{r_d}}$$

Equivalence of **T** Components: Tricky!

Logical relations: related inputs to related outputs

$$\mathcal{V}[\![\tau_1 \to \tau_2]\!] = \{(W, \lambda x.e_1, \lambda x.e_1) \mid \ldots\}$$

$$\mathcal{HV}[\![\forall[\Delta].\{\chi;\sigma\}^q]\!] = \{(W, \mathbf{code}[\Delta]\{\chi;\sigma\}^q.\mathbf{I_1}, \mathbf{code}[\Delta]\{\chi;\sigma\}^q.\mathbf{I_2}) \mid \ldots\}$$

Equivalence of **T** Components: Tricky!

Logical relations: related inputs to related outputs

$$\mathcal{V}[\![\tau_1 \to \tau_2]\!] = \{(W, \lambda \mathsf{x}.\mathsf{e}_1, \lambda \mathsf{x}.\mathsf{e}_1) \mid \ldots\}$$

$$\mathcal{HV}[\![\forall[\boldsymbol{\Delta}].\{\boldsymbol{\chi}; \boldsymbol{\sigma}\}^{\mathrm{q}}]\!] = \{(W, \mathbf{code}[\boldsymbol{\Delta}]\{\boldsymbol{\chi}; \boldsymbol{\sigma}\}^{\mathrm{q}}.\mathbf{I_1}, \mathbf{code}[\boldsymbol{\Delta}]\{\boldsymbol{\chi}; \boldsymbol{\sigma}\}^{\mathrm{q}}.\mathbf{I_2}) \mid \ldots\}$$

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= $\mathbf{code}[\Delta]\{\chi;\sigma\}^{q}.\mathbf{I}$

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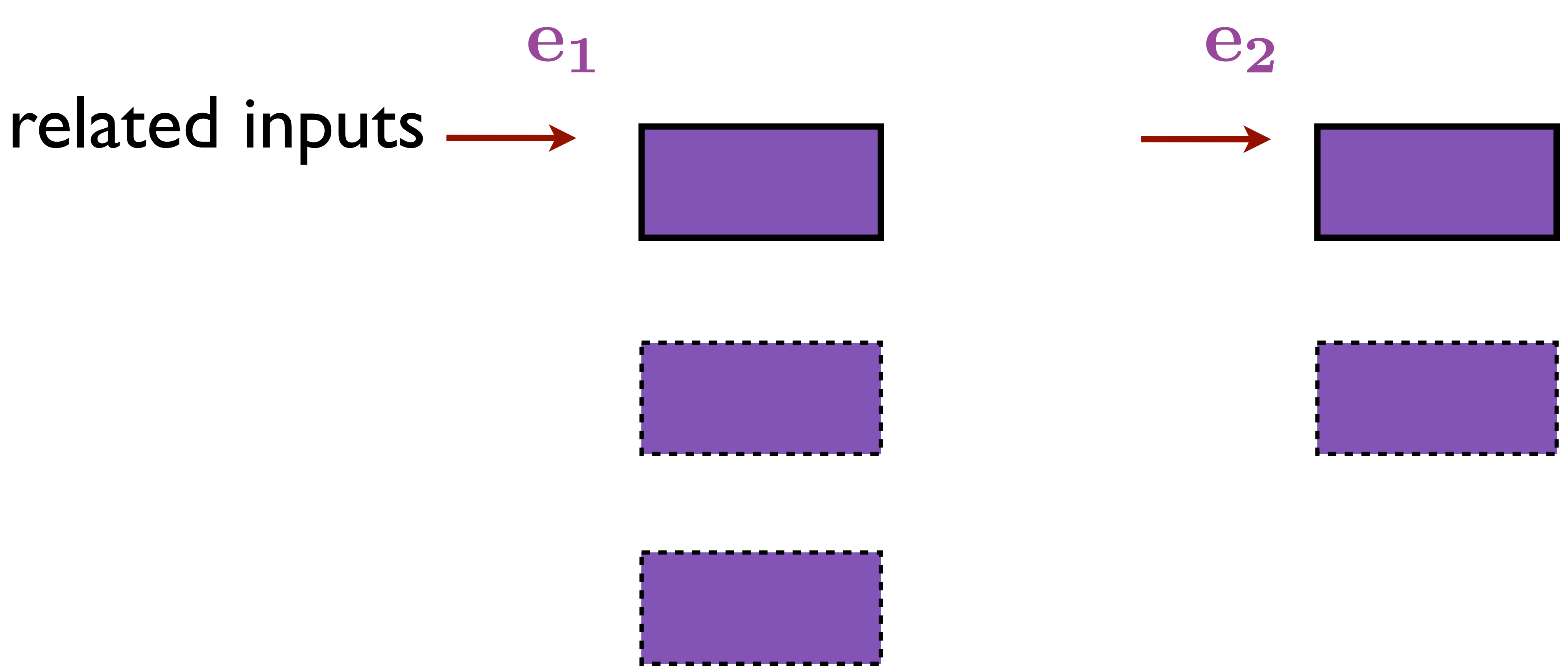

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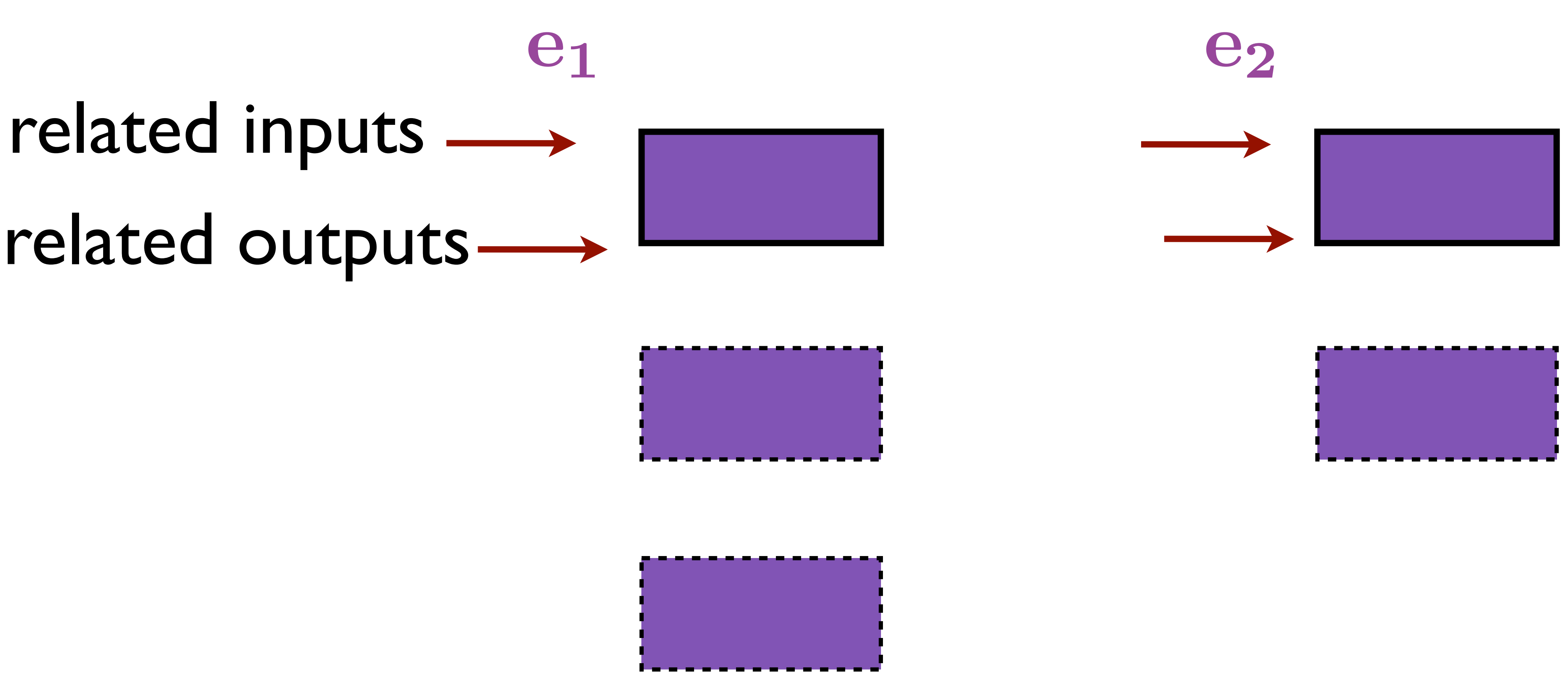

Equivalence of **T** Components: Tricky!

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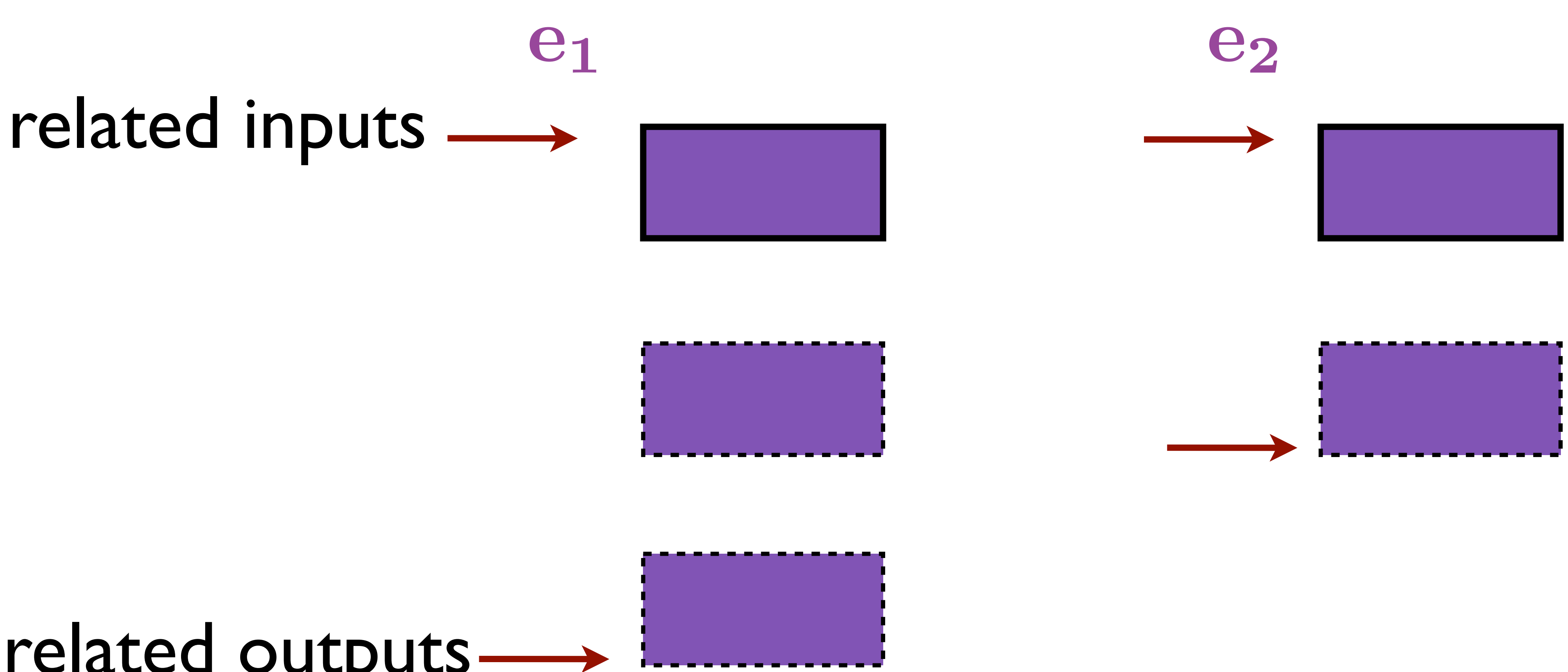

Code Generation: **A** to **T**

$\boxed{\tau^{\mathcal{T}}}$ Type translation

$$
\begin{aligned}
&\mathbf{box}\,\forall[\overline{\alpha}].(\tau_1,\ldots,\tau_n)\rightarrow\tau'^{\mathcal{T}}\\
&\quad=\mathbf{box}\,\forall[\overline{\alpha},\zeta,\epsilon].\\
&\qquad\qquad\{\mathtt{ra}:\mathbf{box}\,\forall[].\{\mathtt{r1}:\tau'^{\mathcal{T}};\zeta\}^{\epsilon};\\
&\qquad\qquad\ \tau_n{}^{\mathcal{T}}::\cdots::\tau_1{}^{\mathcal{T}}::\zeta\}^{\mathtt{ra}}
\end{aligned}
$$

Code Generation: **A** to **T**

$\boxed{\tau^{\mathcal{T}}}$ Type translation

$$\begin{aligned}&\mathbf{box}\,\forall[\overline{\alpha}].(\tau_1,\ldots,\tau_n)\to\tau'^{\mathcal{T}}\\&\quad=\mathrm{box}\,\forall[\overline{\alpha},\zeta,\epsilon].\\&\qquad\qquad\{\mathtt{ra}:\mathrm{box}\,\forall[].\{\mathtt{r1}:\tau'^{\mathcal{T}};\zeta\}^{\epsilon};\\&\qquad\qquad\ \tau_n{}^{\mathcal{T}}::\cdots::\tau_1{}^{\mathcal{T}}::\zeta\}^{\mathtt{ra}}\end{aligned}$$

$\boxed{\Psi;\Delta;\Gamma\vdash \mathbf{e}:\tau\rightsquigarrow \mathrm{e}}$

$$\Psi^{\mathcal{T}};\Delta^{\mathcal{T}};\cdot;\cdot;\Gamma^{\mathcal{T}}::\bullet;\mathrm{end}[\tau^{\mathcal{T}};\Gamma^{\mathcal{T}}::\bullet]\vdash \mathrm{e}:\tau^{\mathcal{T}};\Gamma^{\mathcal{T}}::\bullet$$

Interoperability: **A** and **T**

$$\frac{\Psi; \Delta; \Gamma; \cdot; \sigma; \mathbf{end}[\tau^{\langle \mathcal{T} \rangle}; \sigma'] \vdash \mathrm{e} : \tau^{\langle \mathcal{T} \rangle}; \sigma'}{\Psi; \Delta; \Gamma; \chi; \sigma; \mathbf{out} \vdash {}^{\tau}\mathcal{A}\mathcal{T}\mathrm{e} : \tau; \sigma'}$$

Interoperability: **A** and **T**

$$\frac{\Psi; \Delta; \Gamma; \cdot; \boldsymbol{\sigma}; \mathbf{end}[\boldsymbol{\tau}^{\langle \mathcal{T} \rangle}; \boldsymbol{\sigma}'] \vdash \mathbf{e} : \boldsymbol{\tau}^{\langle \mathcal{T} \rangle}; \boldsymbol{\sigma}'}{\Psi; \Delta; \Gamma; \boldsymbol{\chi}; \boldsymbol{\sigma}; \mathbf{out} \vdash {}^{\boldsymbol{\tau}}\mathcal{A}\mathcal{T}\mathbf{e} : \boldsymbol{\tau}; \boldsymbol{\sigma}'}$$

Interoperability: **A** and **T**

$$\frac{\Psi; \Delta; \Gamma; \cdot; \boldsymbol{\sigma}; \mathbf{end}[\boldsymbol{\tau}^{\langle \mathcal{T} \rangle}; \boldsymbol{\sigma}'] \vdash \mathrm{e} : \boldsymbol{\tau}^{\langle \mathcal{T} \rangle}; \boldsymbol{\sigma}'}{\Psi; \Delta; \Gamma; \chi; \boldsymbol{\sigma}; \mathbf{out} \vdash {}^{\boldsymbol{\tau}}\mathcal{AT}\,\mathrm{e} : \boldsymbol{\tau}; \boldsymbol{\sigma}'}$$

$$\frac{{}^{\boldsymbol{\tau}}\mathbf{AT}(M.\mathbf{M.R(r)}, M) = (\mathsf{v}, M')}{\langle M \mid E[{}^{\boldsymbol{\tau}}\mathcal{AT}\,\mathtt{ret}\ \mathbf{end}[\boldsymbol{\tau}^{\langle \mathcal{T} \rangle}; \boldsymbol{\sigma}], \mathbf{r}] \rangle \longmapsto \langle M' \mid E[\mathsf{v}] \rangle}$$

Interoperability: **A** and **T**

$$\frac{\Psi; \Delta; \Gamma; \cdot; \boldsymbol{\sigma}; \mathbf{end}[\boldsymbol{\tau}^{\langle \mathcal{T} \rangle}; \boldsymbol{\sigma}'] \vdash \mathrm{e} : \boldsymbol{\tau}^{\langle \mathcal{T} \rangle}; \boldsymbol{\sigma}'}{\Psi; \Delta; \Gamma; \chi; \boldsymbol{\sigma}; \mathbf{out} \vdash {}^{\boldsymbol{\tau}}\mathcal{A}\mathcal{T}\mathrm{e} : \boldsymbol{\tau}; \boldsymbol{\sigma}'}$$

$$\frac{{}^{\boldsymbol{\tau}}\mathbf{AT}(M.\mathbf{M.R(r)}, M) = (\mathsf{v}, M')}{\langle M \mid E[{}^{\boldsymbol{\tau}}\mathcal{A}\mathcal{T}\mathtt{ret}\,\mathbf{end}[\boldsymbol{\tau}^{\langle \mathcal{T} \rangle}; \boldsymbol{\sigma}], \mathbf{r}]\rangle \longmapsto \langle M' \mid E[\mathsf{v}]\rangle}$$

Interoperability: **A** and **T**

$$\iota \quad ::= \cdots \mid \texttt{import}\ \mathbf{r_d}, {}^{\sigma}\mathcal{TA}^{\tau}\,\mathbf{e}$$

$$\frac{\mathbf{TA}^{\tau}(\mathsf{v}, M) = (\mathbf{w}, M')}{\langle M \mid E[\texttt{import}\ \mathbf{r_d}, {}^{\sigma'}\mathcal{TA}^{\tau}\,\mathsf{v}; \mathbf{I}]\rangle \longmapsto \langle M' \mid E[\texttt{mv}\ \mathbf{r_d}, \mathbf{w}; \mathbf{I}]\rangle}$$

$$\frac{\begin{array}{c}\sigma = \tau_0 :: \cdots :: \tau_j :: \sigma_0 \qquad \sigma' = \tau'_0 :: \cdots :: \tau'_k :: \sigma_0 \\ \Psi; \Delta, \zeta; \Gamma; \chi; (\tau_0 :: \cdots :: \tau_j :: \zeta); \mathrm{out} \vdash \mathbf{e} : \tau; (\tau'_0 :: \cdots :: \tau'_k :: \zeta) \qquad \mathbf{q} = \mathbf{i} > \mathbf{j} \text{ or } \mathbf{q} =\end{array}}{\Psi; \Delta; \Gamma; \chi; \sigma; \mathbf{q} \vdash \texttt{import}\ \mathbf{r_d}, {}^{\sigma_0}\mathcal{TA}^{\tau}\,\mathbf{e} \Rightarrow \Delta; (\mathbf{r_d} : \tau^{\mathcal{T}}); \sigma'; \mathrm{inc}(\mathbf{q}, \mathbf{k}-\mathbf{j})}$$

Interoperability: **A** and **T**

$$\iota \quad ::= \cdots \mid \texttt{import}\ \mathbf{r_d},{}^{\sigma}\mathcal{T}\mathcal{A}^{\tau}\,\mathbf{e}$$

$$\frac{\mathbf{TA}^{\tau}(\mathsf{v}, M) = (\mathbf{w}, M')}{\langle M \mid E[\texttt{import}\ \mathbf{r_d},{}^{\sigma'}\mathcal{T}\mathcal{A}^{\tau}\,\mathsf{v};\mathbf{I}]\rangle \longmapsto \langle M' \mid E[\texttt{mv}\ \mathbf{r_d},\mathbf{w};\mathbf{I}]\rangle}$$

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Other Issues

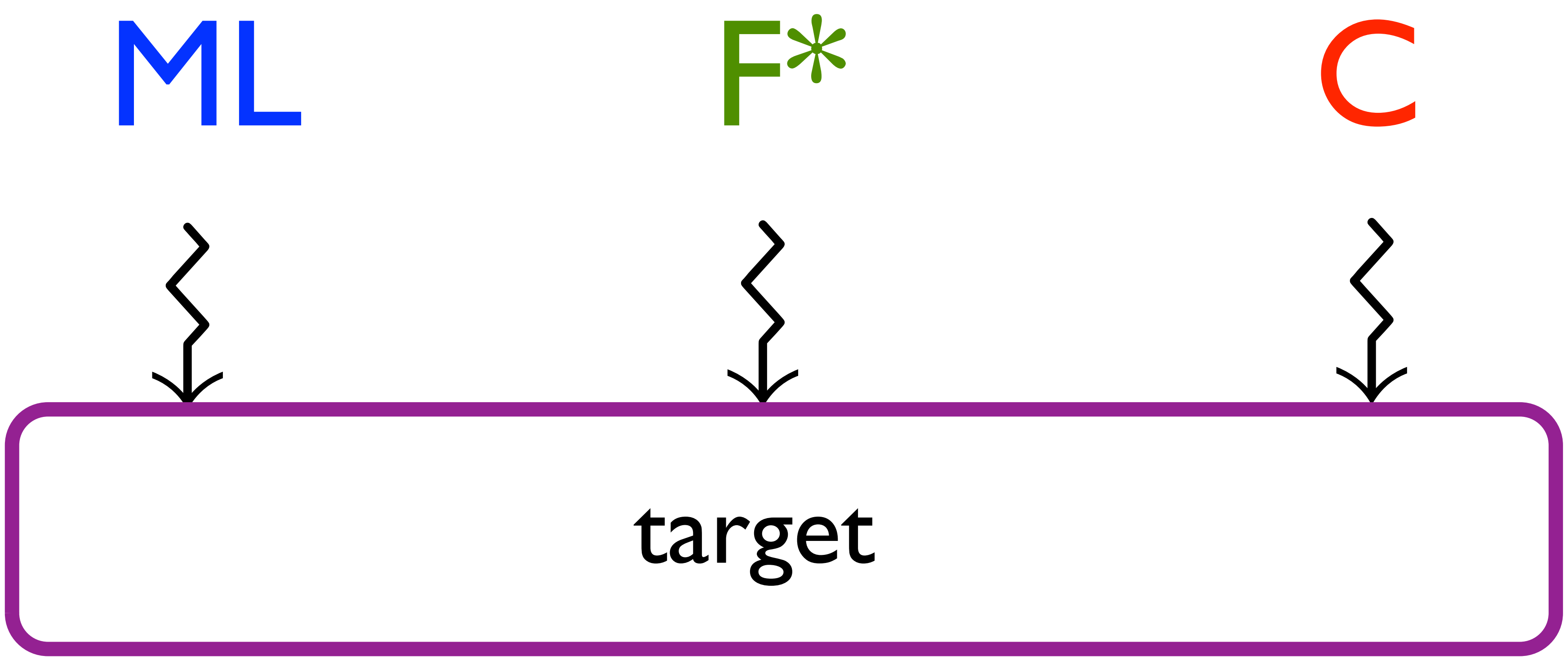
Contexts of **FCAT**

- plugging **T** context with a component is subtle

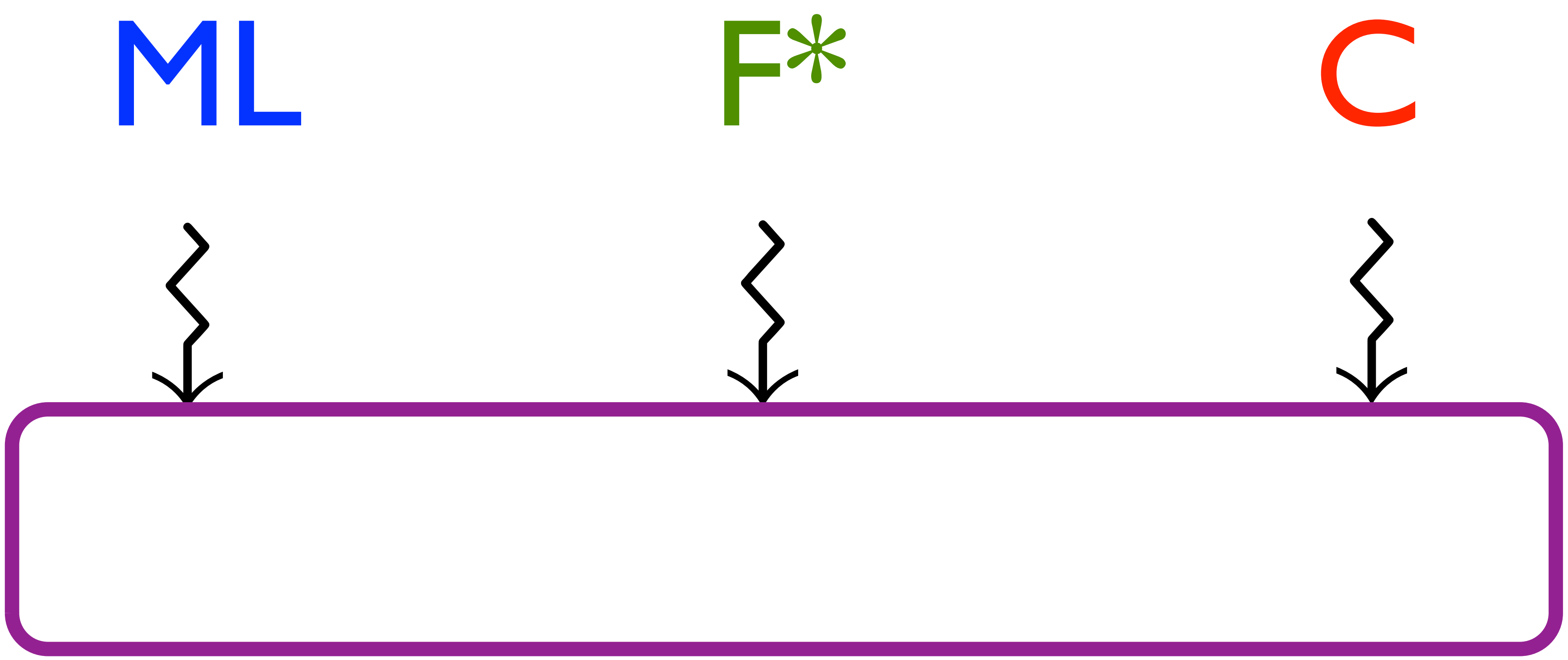
$$\begin{aligned} \mathbf{C} &::= (\mathbf{C_I}, \mathbf{H}) \mid (\mathbf{I}, \mathbf{C_H}) \\ \mathbf{C_I} &::= [\cdot] \mid \iota; \mathbf{C_I} \mid \texttt{import}\ \mathbf{r_d}, {}^{\sigma}\mathcal{T}\mathcal{A}^{\tau}\ \mathsf{C}; \mathbf{I} \\ \mathbf{C_H} &::= \mathbf{C_H}, \ell \mapsto \mathbf{h} \mid \mathbf{H}, \ell \mapsto \mathbf{code}[\Delta]\{\chi; \sigma\}^{\mathbf{q}}.\mathbf{C_I} \end{aligned}$$

Logical Relation for **FCAT** .... nontrivial!

Stepping Back... where's this going?



Stepping Back... where's this going?



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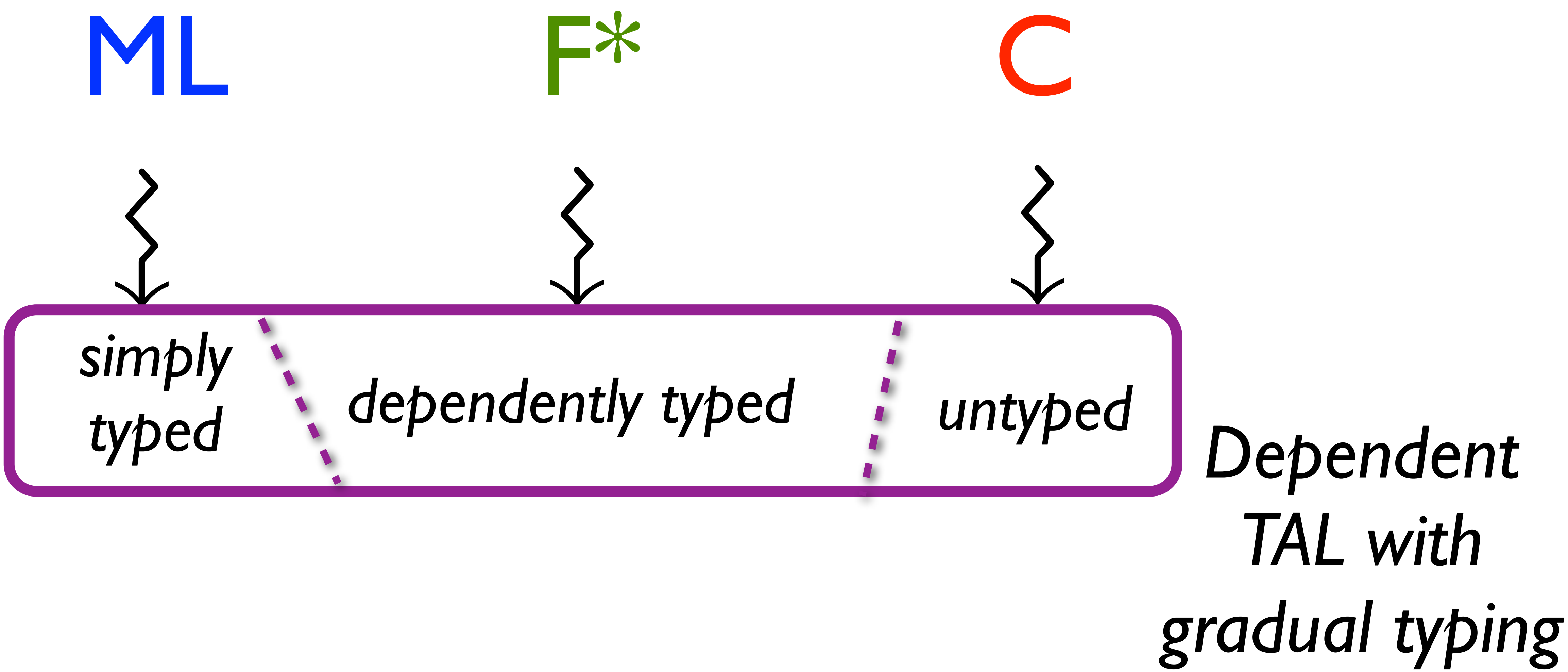

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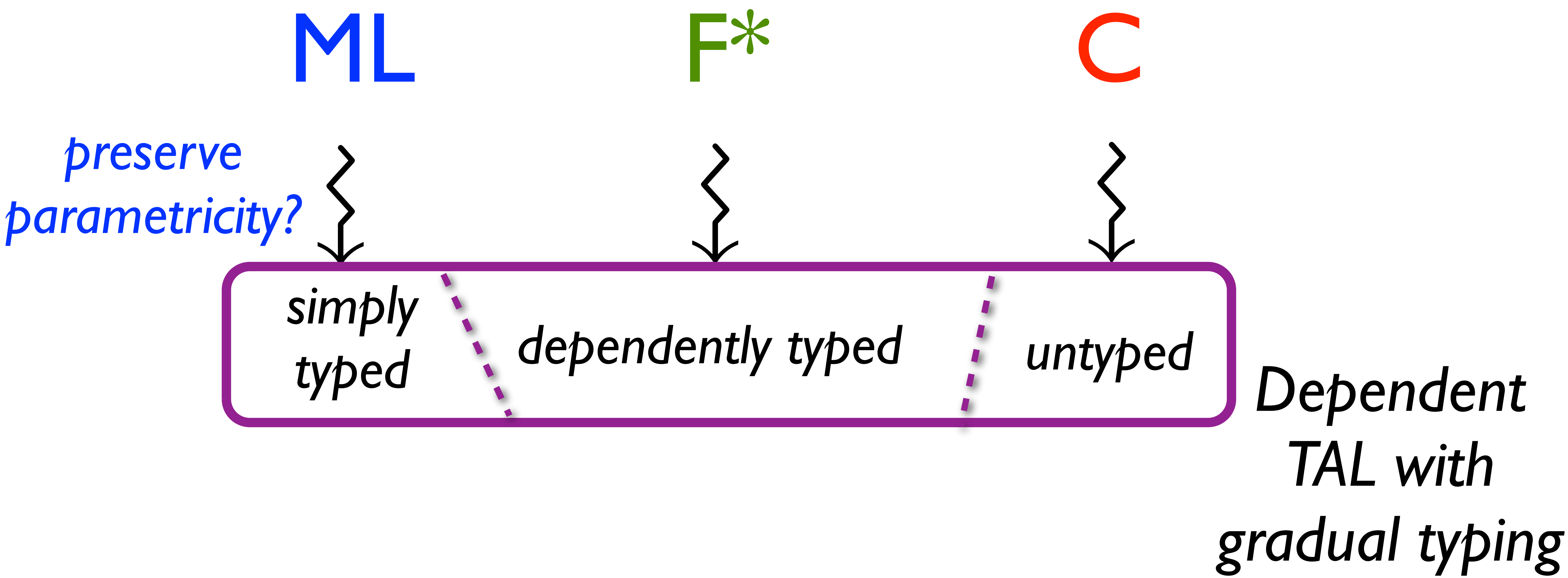

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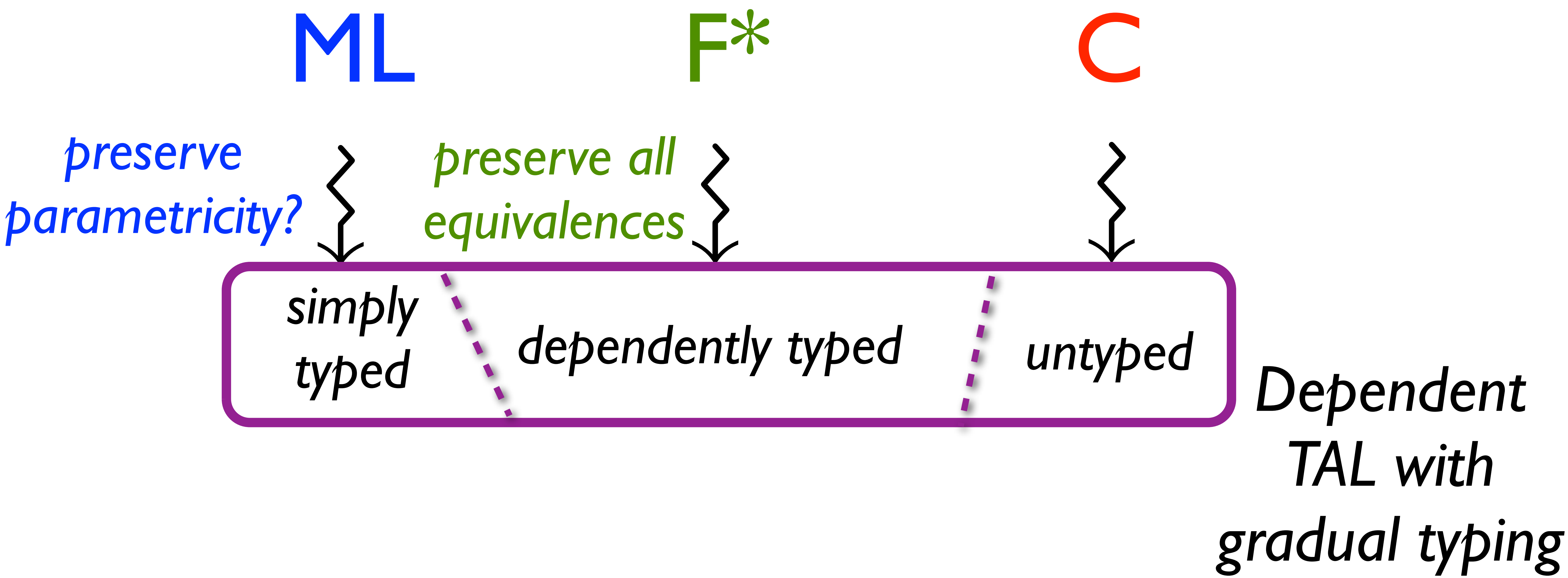
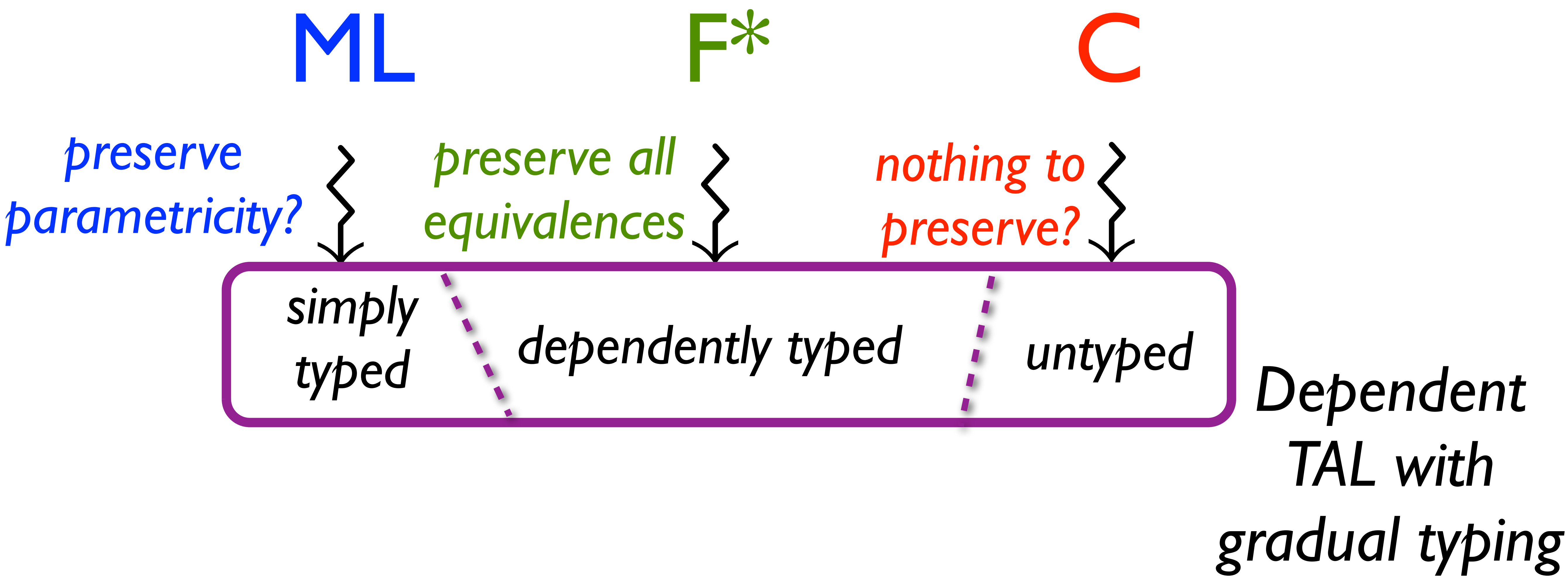
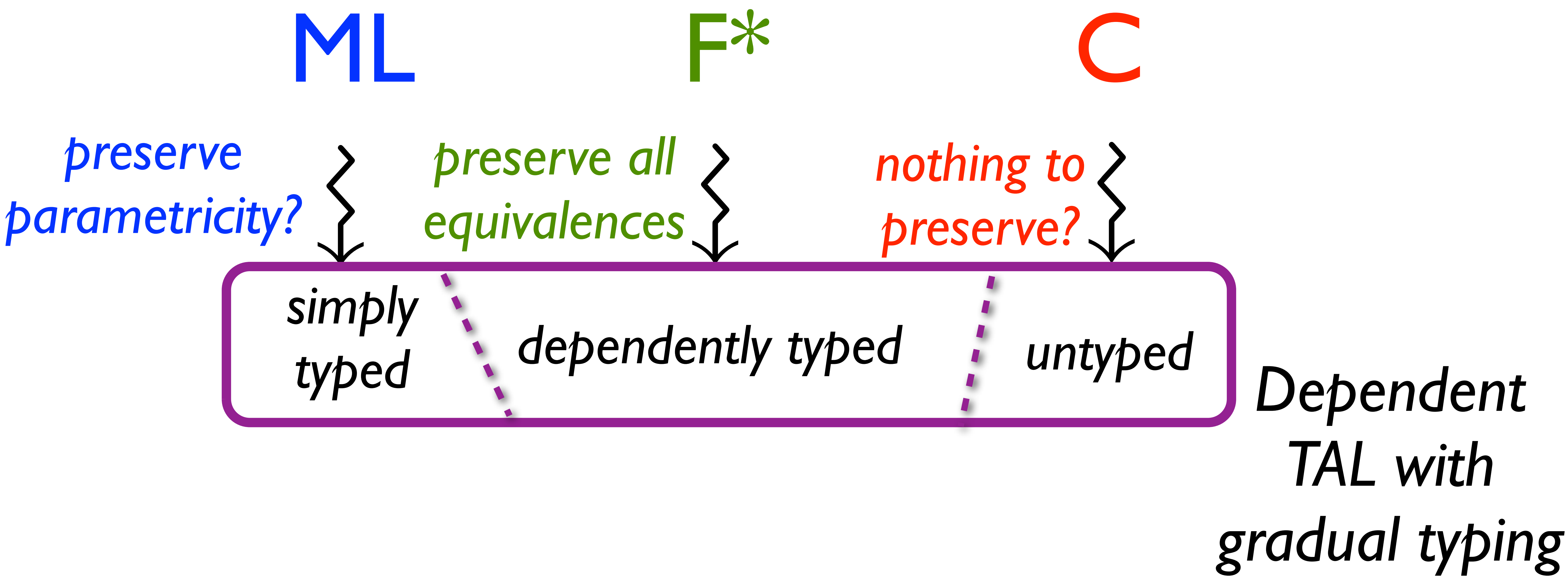


Stepping Back... where's this going?



Stepping Back... where's this going?



It's about principled language interoperability!

Conclusions

Correct compilation of components, not just whole programs

- it's a language interoperability problem!

Multi-language approach:

- works for multi-pass compilers
- supports linking with target code of arbitrary provenance
- an opportunity to study principled interoperability
- interoperability semantics provides a specification of when source and target are related
- but have to get all the languages to fit together!

Questions?