

Errata: The Complexity of Valued Constraint Satisfaction Problems

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1 Errata

- Page 23, line 2 (second line of Definition 1.10): change $\langle f_1, f_1 \rangle$ to $\langle r_1, f_1 \rangle$. (Thanks to Hiroshi Hirai.)
- Page 24, Figure 1.5: subscript k should be n on the last line of Figure 1.5 (three times):
 - change $\mathbf{x}'_n = f_k(\mathbf{x}_1, \dots, \mathbf{x}_k)$ to $\mathbf{x}'_n = f_n(\mathbf{x}_1, \dots, \mathbf{x}_k)$.
 - change $\mathbf{x}'_k[2]$ to $\mathbf{x}'_n[2]$.
 - change $\mathbf{x}'_k[m]$ to $\mathbf{x}'_n[m]$.
- Page 24, Figure 1.5: the lower sum goes from $i = 1$ to n , not k . Thus,

$$\sum_{i=1}^k r_i \phi(\mathbf{x}'_i)$$

should be

$$\sum_{i=1}^n r_i \phi(\mathbf{x}'_i).$$

2 Comments

The material covered in Chapter 2 has been published in [1] and the material covered in Chapter 7 has been published in [2].

3 Open Problems

Chapter 10 of [3] lists the following open problems:

1. What is the structure of weighted clones?
2. Which valued constraint languages are tractable?
3. Which fractional polymorphisms guarantee tractability?
4. Are all tractable languages solvable by the basic LP relaxation?
5. What are the extreme rays of (k -ary) submodular functions?
6. Which functions are expressible by binary submodular functions?
7. Which classes of submodular functions are closed under expressibility?
8. Which classes of VCSPs are captured by subproblems on k -variables?
9. What other hybrid classes of VCSPs are there?

10. What other optimisation problems are tractable (with respect to exact solvability and other notions of tractability) for functions captured by fractional polymorphisms (for both explicitly given functions and functions given by oracles)?

Below is a list of known answers:

- The complexity of finite-valued languages has been completely determined: they are all solvable by BLP [4] and characterised by admitting a symmetric binary fractional polymorphism [5]. This answers Questions 2, 3, and 4 for finite-valued languages.
- One new direction related to Question 9 has been considered in [6].
- Skew bisubmodular functions can be minimised exactly in polynomial time [7, 8] and k -submodular functions can be approximately maximised [9].

References

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